The rich sound of a piano is due to waves on strings that are under tension. Many such strings can be seen in this photograph. Waves also travel on the soundboard, which is visible below the strings. In this chapter, we study the fundamental principles of wave phenomena. (Kathy Ferguson Johnson/PhotoEdit/PictureQuest)
Most of us experienced waves as children when we dropped a pebble into a pond. At the point where the pebble hits the water’s surface, waves are created. These waves move outward from the creation point in expanding circles until they reach the shore. If you were to examine carefully the motion of a beach ball floating on the disturbed water, you would see that the ball moves vertically and horizontally about its original position but does not undergo any net displacement away from or toward the point where the pebble hit the water. The small elements of water in contact with the beach ball, as well as all the other water elements on the pond’s surface, behave in the same way. That is, the water wave moves from the point of origin to the shore, but the water is not carried with it.

The world is full of waves, the two main types being mechanical waves and electromagnetic waves. In the case of mechanical waves, some physical medium is being disturbed—in our pebble and beach ball example, elements of water are disturbed. Electromagnetic waves do not require a medium to propagate; some examples of electromagnetic waves are visible light, radio waves, television signals, and x-rays. Here, in this part of the book, we study only mechanical waves.

The wave concept is abstract. When we observe what we call a water wave, what we see is a rearrangement of the water’s surface. Without the water, there would be no wave. A wave traveling on a string would not exist without the string. Sound waves could not travel from one point to another if there were no air molecules between the two points. With mechanical waves, what we interpret as a wave corresponds to the propagation of a disturbance through a medium.

Considering further the beach ball floating on the water, note that we have caused the ball to move at one point in the water by dropping a pebble at another location. The ball has gained kinetic energy from our action, so energy must have transferred from the point at which we drop the pebble to the position of the ball. This is a central feature of wave motion—energy is transferred over a distance, but matter is not.

All waves carry energy, but the amount of energy transmitted through a medium and the mechanism responsible for that transport of energy differ from case to case. For instance, the power of ocean waves during a storm is much greater than the power of sound waves generated by a single human voice.

16.1 Propagation of a Disturbance

In the introduction, we alluded to the essence of wave motion—the transfer of energy through space without the accompanying transfer of matter. In the list of energy transfer mechanisms in Chapter 7, two mechanisms depend on waves—mechanical waves and electromagnetic radiation. By contrast, in another mechanism—matter transfer—the energy transfer is accompanied by a movement of matter through space.

All mechanical waves require (1) some source of disturbance, (2) a medium that can be disturbed, and (3) some physical mechanism through which elements of the medium can influence each other. One way to demonstrate wave
motion is to flick one end of a long rope that is under tension and has its opposite end
fixed, as shown in Figure 16.1. In this manner, a single bump (called a pulse) is formed
and travels along the rope with a definite speed. Figure 16.1 represents four consecu-
tive “snapshots” of the creation and propagation of the traveling pulse. The rope is the
medium through which the pulse travels. The pulse has a definite height and a definite
speed of propagation along the medium (the rope). As we shall see later, the properties
of this particular medium that determine the speed of the disturbance are the ten-
sion in the rope and its mass per unit length. The shape of the pulse changes very little
as it travels along the rope.\footnote{In reality, the pulse changes shape and gradually spreads out during the motion. This effect is
called dispersion and is common to many mechanical waves as well as to electromagnetic waves. We do not consider dispersion in this chapter.}

We shall first focus our attention on a pulse traveling through a medium. Once we
have explored the behavior of a pulse, we will then turn our attention to a wave, which is
a periodic disturbance traveling through a medium. We created a pulse on our rope by
flicking the end of the rope once, as in Figure 16.1. If we were to move the end of the
rope up and down repeatedly, we would create a traveling wave, which has characteris-
tics that a pulse does not have. We shall explore these characteristics in Section 16.2.

As the pulse in Figure 16.1 travels, each disturbed element of the rope moves in a
direction \textit{perpendicular} to the direction of propagation. Figure 16.2 illustrates this point
for one particular element, labeled $P$. Note that no part of the rope ever moves in the
direction of the propagation.

A traveling wave or pulse that causes the elements of the disturbed medium to move
perpendicular to the direction of propagation is called a \textbf{transverse wave}.

Figure 16.2 A transverse pulse traveling on a stretched rope. The direction of motion of any element $P$ of the rope (blue arrows) is per-
pendicular to the direction of propagation (red arrows).

Compare this with another type of pulse—one moving down a long, stretched
spring, as shown in Figure 16.3. The left end of the spring is pushed briefly to the right
and then pulled briefly to the left. This movement creates a sudden compression of a
region of the coils. The compressed region travels along the spring (to the right in Fig-
ure 16.3). The compressed region is followed by a region where the coils are extended.
Notice that the direction of the displacement of the coils is parallel to the direction of
propagation of the compressed region.

A traveling wave or pulse that causes the elements of the medium to move parallel
to the direction of propagation is called a \textbf{longitudinal wave}.

Figure 16.3 A longitudinal pulse along a stretched spring. The displacement of the coils is parallel to the direction of the propagation.

Sound waves, which we shall discuss in Chapter 17, are another example of longitudinal waves. The disturbance in a sound wave is a series of high-pressure and low-
pressure regions that travel through air.

Some waves in nature exhibit a combination of transverse and longitudinal dis-
placements. Surface water waves are a good example. When a water wave travels on the
surface of deep water, elements of water at the surface move in nearly circular paths, as
shown in Figure 16.4. Note that the disturbance has both transverse and longitudinal
components. The transverse displacements seen in Figure 16.4 represent the variations in vertical position of the water elements. The longitudinal displacement can be explained as follows: as the wave passes over the water’s surface, water elements at the highest points move in the direction of propagation of the wave, whereas elements at the lowest points move in the direction opposite the propagation.

The three-dimensional waves that travel out from points under the Earth’s surface along a fault at which an earthquake occurs are of both types—transverse and longitudinal. The longitudinal waves are the faster of the two, traveling at speeds in the range of 7 to 8 km/s near the surface. These are called P waves (with “P” standing for primary) because they travel faster than the transverse waves and arrive at a seismograph (a device used to detect waves due to earthquakes) first. The slower transverse waves, called S waves (with “S” standing for secondary), travel through the Earth at 4 to 5 km/s near the surface. By recording the time interval between the arrivals of these two types of waves at a seismograph, the distance from the seismograph to the point of origin of the waves can be determined. A single measurement establishes an imaginary sphere centered on the seismograph, with the radius of the sphere determined by the difference in arrival times of the P and S waves. The origin of the waves is located somewhere on that sphere. The imaginary spheres from three or more monitoring stations located far apart from each other intersect at one region of the Earth, and this region is where the earthquake occurred.

Consider a pulse traveling to the right on a long string, as shown in Figure 16.5. Figure 16.5a represents the shape and position of the pulse at time \( t = 0 \). At this time, the shape of the pulse, whatever it may be, can be represented by some mathematical function which we will write as \( y(x, 0) = f(x) \). This function describes the transverse position \( y \) of the element of the string located at each value of \( x \) at time \( t = 0 \). Because the speed of the pulse is \( v \), the pulse has traveled to the right a distance \( vt \) at the time \( t \) (Fig. 16.5b). We assume that the shape of the pulse does not change with time. Thus, at time \( t \), the shape of the pulse is the same as it was at time \( t = 0 \), as in Figure 16.5a.
Chapter 16: Wave Motion

Consequently, an element of the string at \( x \) at this time has the same \( y \) position as an element located at \( x - vt \) had at time \( t = 0 \):

\[
y(x, t) = y(x - vt, 0)
\]

In general, then, we can represent the transverse position \( y \) for all positions and times, measured in a stationary frame with the origin at \( O \), as

\[
y(x, t) = f(x - vt) \quad (16.1)
\]

Similarly, if the pulse travels to the left, the transverse positions of elements of the string are described by

\[
y(x, t) = f(x + vt) \quad (16.2)
\]

The function \( y \), sometimes called the wave function, depends on the two variables \( x \) and \( t \). For this reason, it is often written \( y(x, t) \), which is read “\( y \) as a function of \( x \) and \( t \).”

It is important to understand the meaning of \( y \). Consider an element of the string at point \( P \), identified by a particular value of its \( x \) coordinate. As the pulse passes through \( P \), the \( y \) coordinate of this element increases, reaches a maximum, and then decreases to zero. **The wave function \( y(x, t) \) represents the \( y \) coordinate—the transverse position—of any element located at position \( x \) at any time \( t \).** Furthermore, if \( t \) is fixed (as, for example, in the case of taking a snapshot of the pulse), then the wave function \( y(x) \), sometimes called the waveform, defines a curve representing the actual geometric shape of the pulse at that time.

**Quick Quiz 16.1** In a long line of people waiting to buy tickets, the first person leaves and a pulse of motion occurs as people step forward to fill the gap. As each person steps forward, the gap moves through the line. Is the propagation of this gap (a) transverse (b) longitudinal?

**Quick Quiz 16.2** Consider the “wave” at a baseball game: people stand up and shout as the wave arrives at their location, and the resultant pulse moves around the stadium. Is this wave (a) transverse (b) longitudinal?

---

**Example 16.1 A Pulse Moving to the Right**

A pulse moving to the right along the \( x \) axis is represented by the wave function

\[
y(x, t) = \frac{2}{(x - 3.0t)^2 + 1}
\]

where \( x \) and \( y \) are measured in centimeters and \( t \) is measured in seconds. Plot the wave function at \( t = 0 \), \( t = 1.0 \) s, and \( t = 2.0 \) s.

**Solution** First, note that this function is of the form \( y = f(x - vt) \). By inspection, we see that the wave speed is \( v = 3.0 \) cm/s. Furthermore, the maximum value of \( y \) is given by \( A = 2.0 \) cm. (We find the maximum value of the function representing \( y \) by letting \( x - 3.0t = 0 \).) The wave function expressions are

\[
y(x, 0) = \frac{2}{x^2 + 1} \quad \text{at } t = 0
\]

\[
y(x, 1.0) = \frac{2}{(x - 3.0)^2 + 1} \quad \text{at } t = 1.0 \text{ s}
\]

We now use these expressions to plot the wave function versus \( x \) at these times. For example, let us evaluate \( y(x, 0) \) at \( x = 0.50 \) cm:

\[
y(0.50, 0) = \frac{2}{(0.50)^2 + 1} = 1.6 \text{ cm}
\]

Likewise, at \( x = 1.0 \) cm, \( y(1.0, 0) = 1.0 \) cm, and at \( y = 2.0 \) cm, \( y(2.0, 0) = 0.40 \) cm. Continuing this procedure for other values of \( x \) yields the wave function shown in Figure 16.6a. In a similar manner, we obtain the graphs of \( y(x, 1.0) \) and \( y(x, 2.0) \), shown in Figure 16.6b and c, respectively. These snapshots show that the pulse moves to the right without changing its shape and that it has a constant speed of 3.0 cm/s.

**What If? (A)** What if the wave function were

\[
y(x, t) = \frac{2}{(x + 3.0t)^2 + 1}
\]
16.2 Sinusoidal Waves

In this section, we introduce an important wave function whose shape is shown in Figure 16.7. The wave represented by this curve is called a sinusoidal wave because the curve is the same as that of the function \( \sin \theta \) plotted against \( \theta \). On a rope, a sinusoidal wave could be established by shaking the end of the rope up and down in simple harmonic motion.

The sinusoidal wave is the simplest example of a periodic continuous wave and can be used to build more complex waves (see Section 18.8). The brown curve in Figure 16.7 represents a snapshot of a traveling sinusoidal wave at \( t = 0 \), and the blue curve represents a snapshot of the wave at some later time \( t \). Notice two types of motion that can be seen in your mind. First, the entire waveform in Figure 16.7 moves to the right, so that the brown curve moves toward the right and eventually reaches the position of the blue curve. This is the motion of the wave. If we focus on one element of the medium, such as the element at \( x = 0 \), we see that each element moves up and down along the \( y \) axis in simple harmonic motion. This is the motion of the elements of the medium. It is important to differentiate between the motion of the wave and the motion of the elements of the medium.

Figure 16.8a shows a snapshot of a wave moving through a medium. Figure 16.8b shows a graph of the position of one element of the medium as a function of time. The
point at which the displacement of the element from its normal position is highest is called the crest of the wave. The distance from one crest to the next is called the wavelength \( \lambda \) (Greek lambda). More generally, the wavelength is the minimum distance between any two identical points (such as the crests) on adjacent waves, as shown in Figure 16.8a.

If you count the number of seconds between the arrivals of two adjacent crests at a given point in space, you are measuring the period \( T \) of the waves. In general, the period is the time interval required for two identical points (such as the crests) of adjacent waves to pass by a point. The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium.

The same information is more often given by the inverse of the period, which is called the frequency \( f \). In general, the frequency of a periodic wave is the number of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval. The frequency of a sinusoidal wave is related to the period by the expression

\[
f = \frac{1}{T}
\]

(16.3)

The frequency of the wave is the same as the frequency of the simple harmonic oscillation of one element of the medium. The most common unit for frequency, as we learned in Chapter 15, is second\(^{-1} \), or hertz (Hz). The corresponding unit for \( T \) is seconds.

The maximum displacement from equilibrium of an element of the medium is called the amplitude \( A \) of the wave.

Waves travel with a specific speed, and this speed depends on the properties of the medium being disturbed. For instance, sound waves travel through room-temperature air with a speed of about 343 m/s (781 mi/h), whereas they travel through most solids with a speed greater than 345 m/s.

Consider the sinusoidal wave in Figure 16.8a, which shows the position of the wave at \( t = 0 \). Because the wave is sinusoidal, we expect the wave function at this instant to be expressed as \( y(x, 0) = A \sin ax \), where \( A \) is the amplitude and \( a \) is a constant to be determined. At \( x = 0 \), we see that \( y(0, 0) = A \sin a(0) = 0 \), consistent with Figure 16.8a. The next value of \( x \) for which \( y \) is zero is \( x = \lambda/2 \). Thus,

\[
y \left( \frac{\lambda}{2}, 0 \right) = A \sin a \left( \frac{\lambda}{2} \right) = 0
\]

For this to be true, we must have \( a(\lambda/2) = \pi \), or \( a = 2\pi/\lambda \). Thus, the function describing the positions of the elements of the medium through which the sinusoidal wave is traveling can be written

\[
y(x, 0) = A \sin \left( \frac{2\pi}{\lambda} x \right)
\]

(16.4)

where the constant \( A \) represents the wave amplitude and the constant \( \lambda \) is the wavelength. We see that the vertical position of an element of the medium is the same whenever \( x \) is increased by an integral multiple of \( \lambda \). If the wave moves to the right with a speed \( v \), then the wave function at some later time \( t \) is

\[
y(x, t) = A \sin \left[ \frac{2\pi}{\lambda} \left( x - vt \right) \right]
\]

(16.5)

That is, the traveling sinusoidal wave moves to the right a distance \( vt \) in the time \( t \), as shown in Figure 16.7. Note that the wave function has the form \( f(x - vt) \) (Eq. 16.1). If the wave were traveling to the left, the quantity \( x - vt \) would be replaced by \( x + vt \), as we learned when we developed Equations 16.1 and 16.2.
By definition, the wave travels a distance of one wavelength in one period $T$. Therefore, the wave speed, wavelength, and period are related by the expression

$$v = \frac{\lambda}{T} \quad (16.6)$$

Substituting this expression for $v$ into Equation 16.5, we find that

$$y = A \sin \left[ 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \right] \quad (16.7)$$

This form of the wave function shows the periodic nature of $y$. (We will often use $y$ rather than $y(x, t)$ as a shorthand notation.) At any given time $t$, $y$ has the same value at the positions $x, x + \lambda, x + 2\lambda$, and so on. Furthermore, at any given position $x$, the value of $y$ is the same at times $t, t + T, t + 2T,$ and so on.

We can express the wave function in a convenient form by defining two other quantities, the angular wave number $k$ (usually called simply the wave number) and the angular frequency $\omega$:

$$k = \frac{2\pi}{\lambda} \quad (16.8)$$

$$\omega = \frac{2\pi}{T} \quad (16.9)$$

Using these definitions, we see that Equation 16.7 can be written in the more compact form

$$y = A \sin(kx - \omega t) \quad (16.10)$$

Using Equations 16.3, 16.8, and 16.9, we can express the wave speed $v$ originally given in Equation 16.6 in the alternative forms

$$v = \frac{\omega}{k} \quad (16.11)$$

$$v = \lambda f \quad (16.12)$$

The wave function given by Equation 16.10 assumes that the vertical position $y$ of an element of the medium is zero at $x = 0$ and $t = 0$. This need not be the case. If it is not, we generally express the wave function in the form

$$y = A \sin(kx - \omega t + \phi) \quad (16.13)$$

where $\phi$ is the phase constant, just as we learned in our study of periodic motion in Chapter 15. This constant can be determined from the initial conditions.

**Quick Quiz 16.3** A sinusoidal wave of frequency $f$ is traveling along a stretched string. The string is brought to rest, and a second traveling wave of frequency $2f$ is established on the string. The wave speed of the second wave is (a) twice that of the first wave (b) half that of the first wave (c) the same as that of the first wave (d) impossible to determine.

**Quick Quiz 16.4** Consider the waves in Quick Quiz 16.3 again. The wavelength of the second wave is (a) twice that of the first wave (b) half that of the first wave (c) the same as that of the first wave (d) impossible to determine.
Quick Quiz 16.5 Consider the waves in Quick Quiz 16.3 again. The amplitude of the second wave is (a) twice that of the first wave (b) half that of the first wave (c) the same as that of the first wave (d) impossible to determine.

Example 16.2 A Traveling Sinusoidal Wave

A sinusoidal wave traveling in the positive \( x \) direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at \( t = 0 \) and \( x = 0 \) is also 15.0 cm, as shown in Figure 16.9.

(A) Find the wave number \( k \), period \( T \), angular frequency \( \omega \), and speed \( v \) of the wave.

**Solution** Using Equations 16.8, 16.3, 16.9, and 16.12, we find the following:

\[
\begin{align*}
A &= 15.0 \text{ cm} \\
\lambda &= 40.0 \text{ cm} \\
\omega &= 2\pi f = 2\pi(8.00 \text{ s}^{-1}) = 50.3 \text{ rad/s} \\
v &= \lambda f = (40.0 \text{ cm})(8.00 \text{ s}^{-1}) = 320 \text{ cm/s} \\
k &= \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40.0 \text{ cm}} = 0.157 \text{ rad/cm} \\
T &= \frac{1}{f} = \frac{1}{8.00 \text{ s}^{-1}} = 0.125 \text{ s}
\end{align*}
\]

(B) Determine the phase constant \( \phi \), and write a general expression for the wave function.

**Solution** Because \( A = 15.0 \text{ cm} \) and because \( \gamma = 15.0 \text{ cm} \) at \( x = 0 \) and \( t = 0 \), substitution into Equation 16.13 gives

\[
y = 15.0 = (15.0) \sin \phi \quad \text{or} \quad \sin \phi = 1
\]

We may take the principal value \( \phi = \pi/2 \text{ rad} \) (or 90°). Hence, the wave function is of the form

\[
y = A \sin \left( kx - \omega t + \frac{\pi}{2} \right) = A \cos(kx - \omega t)
\]

By inspection, we can see that the wave function must have this form, noting that the cosine function has the same shape as the sine function displaced by 90°. Substituting the values for \( A \), \( k \), and \( \omega \) into this expression, we obtain

\[
y = (15.0 \text{ cm}) \cos(0.157x - 50.3t)
\]

**Figure 16.9** (Example 16.2) A sinusoidal wave of wavelength \( \lambda = 40.0 \text{ cm} \) and amplitude \( A = 15.0 \text{ cm} \). The wave function can be written in the form \( y = A \cos(kx - \omega t) \).

Sinusoidal Waves on Strings

In Figure 16.1, we demonstrated how to create a pulse by jerking a taut string up and down once. To create a series of such pulses—a wave—we can replace the hand with an oscillating blade. If the wave consists of a series of identical waveforms, whatever their shape, the relationships \( f = 1/T \) and \( v = \lambda f \) among speed, frequency, period, and wavelength hold true. We can make more definite statements about the wave function if the source of the waves vibrates in simple harmonic motion. Figure 16.10 represents snapshots of the wave created in this way at intervals of \( T/4 \). Because the end of the blade oscillates in simple harmonic motion, each element of the string, such as that at \( P \), also oscillates vertically with simple harmonic motion. This must be the case because each element follows the simple harmonic motion of the blade. Therefore, every element of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of oscillation of the blade. Note that although each element oscillates in the \( y \) direction, the wave travels in the \( x \) direction with a speed \( v \). Of course, this is the definition of a transverse wave.

If the wave at \( t = 0 \) is as described in Figure 16.10b, then the wave function can be written as

\[
y = A \sin(kx - \omega t)
\]

---

2 In this arrangement, we are assuming that a string element always oscillates in a vertical line. The tension in the string would vary if an element were allowed to move sideways. Such motion would make the analysis very complex.
We can use this expression to describe the motion of any element of the string. An element at point \( P \) (or any other element of the string) moves only vertically, and so its \( x \) coordinate remains constant. Therefore, the \textit{transverse speed} \( v_y \) (not to be confused with the wave speed \( v \)) and the \textit{transverse acceleration} \( a_y \) of elements of the string are

\[
\begin{align*}
v_y &= \frac{dy}{dt}_{x \text{ constant}} = \frac{\partial y}{\partial t} = -\omega A \cos(kx - \omega t) \quad (16.14) \\
a_y &= \frac{dv_y}{dt}_{x \text{ constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A \sin(kx - \omega t) \quad (16.15)
\end{align*}
\]

In these expressions, we must use partial derivatives (see Section 8.5) because \( y \) depends on both \( x \) and \( t \). In the operation \( \partial y/\partial t \), for example, we take a derivative with respect to \( t \) while holding \( x \) constant. The maximum values of the transverse speed and transverse acceleration are simply the absolute values of the coefficients of the cosine and sine functions:

\[
\begin{align*}
v_{y,\text{max}} &= \omega A \quad (16.16) \\
a_{y,\text{max}} &= \omega^2 A \quad (16.17)
\end{align*}
\]

The transverse speed and transverse acceleration of elements of the string do not reach their maximum values simultaneously. The transverse speed reaches its maximum value (\( \omega A \)) when \( y = 0 \), whereas the magnitude of the transverse acceleration reaches its maximum value (\( \omega^2 A \)) when \( y = \pm A \). Finally, Equations 16.16 and 16.17 are identical in mathematical form to the corresponding equations for simple harmonic motion, Equations 15.17 and 15.18.

\section*{Quick Quiz 16.6}

The amplitude of a wave is doubled, with no other changes made to the wave. As a result of this doubling, which of the following statements is correct? (a) The speed of the wave changes. (b) The frequency of the wave changes. (c) The maximum transverse speed of an element of the medium changes. (d) All of these are true. (e) None of these is true.
Example 16.3  A Sinusoidally Driven String

The string shown in Figure 16.10 is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Determine the angular frequency \( \omega \) and wave number \( k \) for this wave, and write an expression for the wave function.

Solution Using Equations 16.3, 16.9, and 16.11, we find that

\[
\omega = \frac{2\pi}{T} = 2\pi f = 2\pi(5.00 \, \text{Hz}) = 31.4 \, \text{rad/s}
\]

\[
k = \frac{\omega}{v} = \frac{31.4 \, \text{rad/s}}{20.0 \, \text{m/s}} = 1.57 \, \text{rad/m}
\]

Because \( A = 12.0 \, \text{cm} = 0.120 \, \text{m} \), we have

\[
y = A \sin(kx - \omega t) = (0.120 \, \text{m}) \sin(1.57x - 31.4t)
\]

16.3 The Speed of Waves on Strings

In this section, we focus on determining the speed of a transverse pulse traveling on a taut string. Let us first conceptually predict the parameters that determine the speed. If a string under tension is pulled sideways and then released, the tension is responsible for accelerating a particular element of the string back toward its equilibrium position. According to Newton’s second law, the acceleration of the element increases with increasing tension. If the element returns to equilibrium more rapidly due to this increased acceleration, we would intuitively argue that the wave speed is greater. Thus, we expect the wave speed to increase with increasing tension.

Likewise, the wave speed should decrease as the mass per unit length of the string increases. This is because it is more difficult to accelerate a massive element of the string than a light element. If the tension in the string is \( T \) and its mass per unit length is \( \mu \) (Greek \( \mu \)), then as we shall show, the wave speed is

\[
v = \sqrt{\frac{T}{\mu}} \quad \text{(16.18)}
\]

First, let us verify that this expression is dimensionally correct. The dimensions of \( T \) are \( \text{ML}/\text{T}^2 \), and the dimensions of \( \mu \) are \( \text{M}/\text{L} \). Therefore, the dimensions of \( T/\mu \) are \( \text{L}^2/\text{T}^2 \); hence, the dimensions of \( \sqrt{T/\mu} \) are \( \text{L}/\text{T} \), the dimensions of speed. No other combination of \( T \) and \( \mu \) is dimensionally correct, and if we assume that these are the only variables relevant to the situation, the speed must be proportional to \( \sqrt{T/\mu} \).

Now let us use a mechanical analysis to derive Equation 16.18. Consider a pulse moving on a taut string to the right with a uniform speed \( v \) measured relative to a stationary frame of reference. Instead of staying in this reference frame, it is more convenient to choose as our reference frame one that moves along with the pulse with the same speed as the pulse, so that the pulse is at rest within the frame. This change of reference frame is permitted because Newton’s laws are valid in either a stationary frame or one that moves with constant velocity. In our new reference frame, all elements of the string move to the left—a given element of the string initially to the right of the pulse moves to the left, rises up and follows the shape of the pulse, and then continues to move to the left. Figure 16.11a shows such an element at the instant it is located at the top of the pulse.

The small element of the string of length \( \Delta s \) shown in Figure 16.11a, and magnified in Figure 16.11b, forms an approximate arc of a circle of radius \( R \). In our moving frame of reference (which is moving to the right at a speed \( v \) along with the pulse), the shaded element is moving to the left with a speed \( v \). This element has a centripetal acceleration equal to \( v^2/R \), which is supplied by components of the force \( T \) whose magnitude is the tension in the string. The force \( T \) acts on both sides of the element and is tangent to the arc, as shown in Figure 16.11b. The horizontal components of \( T \) cancel, and each vertical component \( T \sin \theta \) acts radially toward the center of the arc. Hence, the total

\[
v = \sqrt{\frac{T}{\mu}}
\]
radial force on the element is \(2T \sin \theta\). Because the element is small, \(\theta\) is small, and we can use the small-angle approximation \(\sin \theta \approx \theta\). Therefore, the total radial force is

\[F_r = 2T \sin \theta \approx 2T \theta\]

The element has a mass \(m = \mu \Delta s\). Because the element forms part of a circle and subtends an angle \(2\theta\) at the center, \(\Delta s = R(2\theta)\), we find that

\[m = \mu \Delta s = 2\mu R \theta\]

If we apply Newton’s second law to this element in the radial direction, we have

\[F_r = ma = \frac{mv^2}{R}\]

\[2T \theta = \frac{2\mu R \theta v^2}{R} \quad \rightarrow \quad v = \sqrt{\frac{T}{\mu}}\]

This expression for \(v\) is Equation 16.18.

Notice that this derivation is based on the assumption that the pulse height is small relative to the length of the string. Using this assumption, we were able to use the approximation \(\sin \theta \approx \theta\). Furthermore, the model assumes that the tension \(T\) is not affected by the presence of the pulse; thus, \(T\) is the same at all points on the string. Finally, this proof does not assume any particular shape for the pulse. Therefore, we conclude that a pulse of any shape travels along the string with speed \(v = \sqrt{T/\mu}\) without any change in pulse shape.

**Quick Quiz 16.7** Suppose you create a pulse by moving the free end of a taut string up and down once with your hand beginning at \(t = 0\). The string is attached at its other end to a distant wall. The pulse reaches the wall at time \(t\). Which of the following actions, taken by itself, decreases the time interval that it takes for the pulse to reach the wall? More than one choice may be correct. (a) moving your hand more quickly, but still only up and down once by the same amount (b) moving your hand more slowly, but still only up and down once by the same amount (c) moving your hand a greater distance up and down in the same amount of time (d) moving your hand a lesser distance up and down in the same amount of time (e) using a heavier string of the same length and under the same tension (f) using a lighter string of the same length and under the same tension (g) using a string of the same linear mass density but under decreased tension (h) using a string of the same linear mass density but under increased tension
Example 16.4  The Speed of a Pulse on a Cord

A uniform cord has a mass of 0.300 kg and a length of 6.00 m (Fig. 16.12). The cord passes over a pulley and supports a 2.00-kg object. Find the speed of a pulse traveling along this cord.

Solution The tension \( T \) in the cord is equal to the weight of the suspended 2.00-kg object:

\[
T = mg = (2.00 \text{ kg})(9.80 \text{ m/s}^2) = 19.6 \text{ N}
\]

(This calculation of the tension neglects the small mass of the cord. Strictly speaking, the cord can never be exactly horizontal, and therefore the tension is not uniform.) The mass per unit length \( \mu \) of the cord is

\[
\mu = \frac{m}{L} = \frac{0.300 \text{ kg}}{6.00 \text{ m}} = 0.050 \text{ kg/m}
\]

Therefore, the wave speed is

\[
v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{19.6 \text{ N}}{0.050 \text{ kg/m}}} = 19.8 \text{ m/s}
\]

What If? What if the block were swinging back and forth between maximum angles of \( \pm 20^\circ \) with respect to the vertical? What range of wave speeds would this create on the horizontal cord?

Answer Figure 16.13 shows the swinging block at three positions—its highest position, its lowest position, and an arbitrary position. Summing the forces on the block in the radial direction when the block is at an arbitrary position, Newton’s second law gives

\[
(1) \quad \sum F = T - mg \cos \theta = m \frac{v_{\text{block}}^2}{L}
\]

where the acceleration of the block is centripetal, \( L \) is the length of the vertical piece of string, and \( v_{\text{block}} \) is the instantaneous speed of the block at the arbitrary position.

Now consider conservation of mechanical energy for the block-Earth system. We define the zero of gravitational potential energy for the system when the block is at its lowest point, point \( \odot \) in Figure 16.13. Equating the mechanical energy of the system when the block is at \( \bullet \) to the mechanical energy when the block is at an arbitrary position \( \odot \), we have,

\[
E_A = E_B
\]

\[
mg h_{\text{max}} = mg h + \frac{1}{2}mv_{\text{block}}^2
\]

\[
\frac{1}{2}mv_{\text{block}}^2 = 2mg(h_{\text{max}} - h)
\]

Substituting this into Equation (1), we find an expression for \( T \) as a function of angle \( \theta \) and height \( h \):

\[
T - mg \cos \theta = \frac{2mg(h_{\text{max}} - h)}{L}
\]

\[
T = mg \left[ \cos \theta + \frac{2}{L}(h_{\text{max}} - h) \right]
\]

The maximum value of \( T \) occurs when \( \theta = 0 \) and \( h = 0 \):

\[
T_{\text{max}} = mg \left[ \cos 0 + \frac{2}{L}(h_{\text{max}} - 0) \right] = mg \left( 1 + \frac{2h_{\text{max}}}{L} \right)
\]

The minimum value of \( T \) occurs when \( h = h_{\text{max}} \) and \( \theta = \theta_{\text{max}} \):

\[
T_{\text{min}} = mg \left[ \cos \theta_{\text{max}} + \frac{2}{L}(h_{\text{max}} - h_{\text{max}}) \right] = mg \cos \theta_{\text{max}}
\]

Now we find the maximum and minimum values of the wave speed \( v \), using the fact that, as we see from Figure 16.13, \( h \) and \( \theta \) are related by \( h = L - L \cos \theta \):

\[
v_{\text{max}} = \sqrt{\frac{T_{\text{max}}}{\mu}} = \sqrt{\frac{mg[1 + (2h_{\text{max}}/L)]}{\mu}}
\]

\[
v_{\text{min}} = \sqrt{\frac{T_{\text{min}}}{\mu}} = \sqrt{\frac{mg[1 + [2(L - L \cos \theta_{\text{max}})/L]]}{\mu}}
\]
Example 16.5  Rescuing the Hiker

An 80.0-kg hiker is trapped on a mountain ledge following a storm. A helicopter rescues the hiker by hovering above him and lowering a cable to him. The mass of the cable is 8.00 kg, and its length is 15.0 m. A chair of mass 70.0 kg is attached to the end of the cable. The hiker attaches himself to the chair, and the helicopter then accelerates upward. Terrified by hanging from the cable in midair, the hiker tries to signal the pilot by sending transverse pulses up the cable. A pulse takes 0.250 s to travel the length of the cable. What is the acceleration of the helicopter?

Solution  To conceptualize this problem, imagine the effect of the acceleration of the helicopter on the cable. The higher the upward acceleration, the larger is the tension in the cable. In turn, the larger the tension, the higher is the speed of pulses on the cable. Thus, we categorize this problem as a combination of one involving Newton’s laws and one involving the speed of pulses on a string. To analyze the problem, we use the time interval for the pulse to travel from the hiker to the helicopter to find the speed of the pulses on the cable:

\[
v = \frac{\Delta x}{\Delta t} = \frac{15.0 \text{ m}}{0.250 \text{ s}} = 60.0 \text{ m/s}
\]

The speed of the pulses on the cable is given by Equation 16.18, which allows us to find the tension in the cable:

\[
v = \sqrt{\frac{T}{\mu}} \quad \rightarrow \quad T = \mu v^2 = \left( \frac{8.00 \text{ kg}}{15.0 \text{ m}} \right) (60.0 \text{ m/s})^2
\]

\[T = 1.92 \times 10^3 \text{ N}\]

Newton’s second law relates the tension in the cable to the acceleration of the hiker and the chair, which is the same as the acceleration of the helicopter:

\[
\sum F = ma \quad \rightarrow \quad T - mg = ma
\]

\[a = \frac{T}{m} - g = \frac{1.92 \times 10^3 \text{ N}}{150.0 \text{ kg}} - 9.80 \text{ m/s}^2
\]

\[a = 3.00 \text{ m/s}^2\]

To finalize this problem, note that a real cable has stiffness in addition to tension. Stiffness tends to return a wire to its original straight-line shape even when it is not under tension. For example, a piano wire straightens if released from a curved shape; package wrapping string does not.

Stiffness represents a restoring force in addition to tension, and increases the wave speed. Consequently, for a real cable, the speed of 60.0 m/s that we determined is most likely associated with a tension lower than 1.92 \times 10^3 N and a correspondingly smaller acceleration of the helicopter.

Investigate this situation at the Interactive Worked Example link at http://www.pse6.com.

16.4 Reflection and Transmission

We have discussed waves traveling through a uniform medium. We now consider how a traveling wave is affected when it encounters a change in the medium. For example, consider a pulse traveling on a string that is rigidly attached to a support at one end as in Figure 16.14. When the pulse reaches the support, a severe change in the medium occurs—the string ends. The result of this change is that the pulse undergoes reflection—that is, the pulse moves back along the string in the opposite direction.

Note that the reflected pulse is inverted. This inversion can be explained as follows. When the pulse reaches the fixed end of the string, the string produces an upward force on the support. By Newton’s third law, the support must exert an equal-magnitude and oppositely directed (downward) reaction force on the string. This downward force causes the pulse to invert upon reflection.

Now consider another case: this time, the pulse arrives at the end of a string that is free to move vertically, as in Figure 16.15. The tension at the free end is maintained because the string is tied to a ring of negligible mass that is free to slide vertically on a smooth post without friction. Again, the pulse is reflected, but this time it is not inverted. When it reaches the post, the pulse exerts a force on the free end of the string, causing the ring to accelerate upward. The ring rises as high as the incoming pulse.

At the Active Figures link at http://www.pse6.com, you can adjust the linear mass density of the string and the transverse direction of the initial pulse.
and then the downward component of the tension force pulls the ring back down. This movement of the ring produces a reflected pulse that is not inverted and that has the same amplitude as the incoming pulse.

Finally, we may have a situation in which the boundary is intermediate between these two extremes. In this case, part of the energy in the incident pulse is reflected and part undergoes transmission—that is, some of the energy passes through the boundary. For instance, suppose a light string is attached to a heavier string, as in Figure 16.16. When a pulse traveling on the light string reaches the boundary between the two, part of the pulse is reflected and inverted and part is transmitted to the heavier string. The reflected pulse is inverted for the same reasons described earlier in the case of the string rigidly attached to a support.

Note that the reflected pulse has a smaller amplitude than the incident pulse. In Section 16.5, we show that the energy carried by a wave is related to its amplitude. According to the principle of the conservation of energy, when the pulse breaks up into a reflected pulse and a transmitted pulse at the boundary, the sum of the energies of these two pulses must equal the energy of the incident pulse. Because the reflected pulse contains only part of the energy of the incident pulse, its amplitude must be smaller.

When a pulse traveling on a heavy string strikes the boundary between the heavy string and a lighter one, as in Figure 16.17, again part is reflected and part is transmitted. In this case, the reflected pulse is not inverted.

In either case, the relative heights of the reflected and transmitted pulses depend on the relative densities of the two strings. If the strings are identical, there is no discontinuity at the boundary and no reflection takes place.

![Active Figure 16.15](http://www.pse6.com)

At the Active Figures link at http://www.pse6.com, you can adjust the linear mass density of the string and the transverse direction of the initial pulse.

![Active Figure 16.16](http://www.pse6.com)

Figure 16.16 (a) A pulse traveling to the right on a light string attached to a heavier string. (b) Part of the incident pulse is reflected (and inverted), and part is transmitted to the heavier string. See Figure 16.17 for an animation available for both figures at the Active Figures link.

![Active Figure 16.17](http://www.pse6.com)

At the Active Figures link at http://www.pse6.com, you can adjust the linear mass densities of the strings and the transverse direction of the initial pulse.

Active Figure 16.17 (a) A pulse traveling to the right on a heavy string attached to a lighter string. (b) The incident pulse is partially reflected and partially transmitted, and the reflected pulse is not inverted.
According to Equation 16.18, the speed of a wave on a string increases as the mass per unit length of the string decreases. In other words, a wave travels more slowly on a heavy string than on a light string if both are under the same tension. The following general rules apply to reflected waves: when a wave or pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), it is inverted upon reflection. When a wave or pulse travels from medium A to medium B and $v_A < v_B$ (that is, when A is denser than B), it is not inverted upon reflection.

16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings

Waves transport energy when they propagate through a medium. We can easily demonstrate this by hanging an object on a stretched string and then sending a pulse down the string, as in Figure 16.18a. When the pulse meets the suspended object, the object is momentarily displaced upward, as in Figure 16.18b. In the process, energy is transferred to the object and appears as an increase in the gravitational potential energy of the object–Earth system. This section examines the rate at which energy is transported along a string. We shall assume a one-dimensional sinusoidal wave in the calculation of the energy transferred.

Consider a sinusoidal wave traveling on a string (Fig. 16.19). The source of the energy is some external agent at the left end of the string, which does work in producing the oscillations. We can consider the string to be a nonisolated system. As the external agent performs work on the end of the string, moving it up and down, energy enters the system of the string and propagates along its length. Let us focus our attention on an element of the string of length $\Delta x$ and mass $\Delta m$. Each such element moves vertically with simple harmonic motion. Thus, we can model each element of the string as a simple harmonic oscillator, with the oscillation in the $y$ direction. All elements have the same angular frequency $\omega$ and the same amplitude $A$. The kinetic energy $K$ associated with a moving particle is $K = \frac{1}{2}mv^2$. If we apply this equation to an element of length $\Delta x$ and mass $\Delta m$, we see that the kinetic energy $\Delta K$ of this element is

$$
\Delta K = \frac{1}{2}(\Delta m)v_y^2
$$

where $v_y$ is the transverse speed of the element. If $\mu$ is the mass per unit length of the string, then the mass $\Delta m$ of the element of length $\Delta x$ is equal to $\mu \Delta x$. Hence, we can express the kinetic energy of an element of the string as

$$
\Delta K = \frac{1}{2}(\mu \Delta x)v_y^2
$$

(16.19)

As the length of the element of the string shrinks to zero, this becomes a differential relationship:

$$
dK = \frac{1}{2}(\mu dx)v_y^2
$$

We substitute for the general transverse speed of a simple harmonic oscillator using Equation 16.14:

$$
dK = \frac{1}{2}\mu [\omega A \cos(kx - \omega t)]^2 dx
$$

$$
= \frac{1}{2}\mu \omega^2 A^2 \cos^2(kx - \omega t) dx
$$

Figure 16.18 (a) A pulse traveling to the right on a stretched string that has an object suspended from it. (b) Energy is transmitted to the suspended object when the pulse arrives.

Figure 16.19 A sinusoidal wave traveling along the x axis on a stretched string. Every element moves vertically, and every element has the same total energy.
If we take a snapshot of the wave at time $t = 0$, then the kinetic energy of a given element is

$$dK = \frac{1}{2} \mu \omega^2 A^2 \cos^2 kx \, dx$$

Let us integrate this expression over all the string elements in a wavelength of the wave, which will give us the total kinetic energy $K_\lambda$ in one wavelength:

$$K_\lambda = \int_0^\lambda dK = \int_0^\lambda \frac{1}{2} \mu \omega^2 A^2 \cos^2 kx \, dx = \frac{1}{2} \mu \omega^2 A^2 \int_0^\lambda \cos^2 kx \, dx$$

$$= \frac{1}{2} \mu \omega^2 A^2 \left[ \frac{1}{2} x + \frac{1}{4k} \sin 2kx \right]_0^\lambda = \frac{1}{2} \mu \omega^2 A^2 \left[ \frac{1}{2} \lambda \right] = \frac{1}{2} \mu \omega^2 A^2 \lambda$$

In addition to kinetic energy, each element of the string has potential energy associated with it due to its displacement from the equilibrium position and the restoring forces from neighboring elements. A similar analysis to that above for the total potential energy $U_\lambda$ in one wavelength will give exactly the same result:

$$U_\lambda = \frac{1}{2} \mu \omega^2 A^2 \lambda$$

The total energy in one wavelength of the wave is the sum of the potential and kinetic energies:

$$E_\lambda = U_\lambda + K_\lambda = \frac{1}{2} \mu \omega^2 A^2 \lambda$$  \hspace{1cm} (16.20)

As the wave moves along the string, this amount of energy passes by a given point on the string during a time interval of one period of the oscillation. Thus, the power, or rate of energy transfer, associated with the wave is

$$\mathcal{P} = \frac{\Delta E}{\Delta t} = \frac{E_\lambda}{T} = \frac{\frac{1}{2} \mu \omega^2 A^2 \lambda}{T} = \frac{1}{2} \mu \omega^2 A^2 \left( \frac{\lambda}{T} \right)$$

$$\mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v$$  \hspace{1cm} (16.21)

This expression shows that the rate of energy transfer by a sinusoidal wave on a string is proportional to (a) the square of the frequency, (b) the square of the amplitude, and (c) the wave speed. In fact: the rate of energy transfer in any sinusoidal wave is proportional to the square of the angular frequency and to the square of the amplitude.

**Quick Quiz 16.8** Which of the following, taken by itself, would be most effective in increasing the rate at which energy is transferred by a wave traveling along a string?
(a) reducing the linear mass density of the string by one half (b) doubling the wavelength of the wave (c) doubling the tension in the string (d) doubling the amplitude of the wave

**Example 16.6 Power Supplied to a Vibrating String**

A taut string for which $\mu = 5.00 \times 10^{-2} \text{ kg/m}$ is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

**Solution** The wave speed on the string is, from Equation 16.18,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80.0 \text{ N}}{5.00 \times 10^{-2} \text{ kg/m}}} = 40.0 \text{ m/s}$$

Because $f = 60.0 \text{ Hz}$, the angular frequency $\omega$ of the sinusoidal waves on the string has the value

$$\omega = 2\pi f = 2\pi(60.0 \text{ Hz}) = 377 \text{ s}^{-1}$$
16.6 The Linear Wave Equation

In Section 16.1 we introduced the concept of the wave function to represent waves traveling on a string. All wave functions $\psi(x, t)$ represent solutions of an equation called the linear wave equation. This equation gives a complete description of the wave motion, and from it one can derive an expression for the wave speed. Furthermore, the linear wave equation is basic to many forms of wave motion. In this section, we derive this equation as applied to waves on strings.

Suppose a traveling wave is propagating along a string that is under a tension $T$. Let us consider one small string element of length $\Delta x$ (Fig. 16.20). The ends of the element make small angles $\theta_A$ and $\theta_B$ with the $x$ axis. The net force acting on the element in the vertical direction is

$$ \sum F_y = T \sin \theta_B - T \sin \theta_A = T (\sin \theta_B - \sin \theta_A) $$

Because the angles are small, we can use the small-angle approximation $\sin \theta \approx \tan \theta$ to express the net force as

$$ \sum F_y \approx T (\tan \theta_B - \tan \theta_A) $$

(16.22)

Imagine undergoing an infinitesimal displacement outward from the end of the rope element in Figure 16.20 along the blue line representing the force $\mathbf{T}$. This displacement has infinitesimal $x$ and $y$ components and can be represented by the vector $d \mathbf{x} + \mathbf{dy}$. The tangent of the angle with respect to the $x$ axis for this displacement is $dy/dx$. Because we are evaluating this tangent at a particular instant of time, we need to express this in partial form as $\partial y/\partial x$. Substituting for the tangents in Equation 16.22 gives

$$ \sum F_y \approx T \left[ \frac{\partial y}{\partial x} \right]_B - \left[ \frac{\partial y}{\partial x} \right]_A $$

(16.23)

We now apply Newton’s second law to the element, with the mass of the element given by $m = \mu \Delta x$:

$$ \sum F_y = ma_y = \mu \Delta x \left( \frac{\partial^2 y}{\partial t^2} \right) $$

(16.24)

Combining Equation 16.23 with Equation 16.24, we obtain

$$ \mu \Delta x \left( \frac{\partial^2 y}{\partial t^2} \right) = T \left[ \frac{\partial y}{\partial x} \right]_B - \left[ \frac{\partial y}{\partial x} \right]_A $$

$$ \frac{\mu \, \partial^2 y}{T \, \partial t^2} = - \frac{\left[ \frac{\partial y}{\partial x} \right]_B - \left[ \frac{\partial y}{\partial x} \right]_A}{\Delta x} $$

(16.25)
The right side of this equation can be expressed in a different form if we note that the partial derivative of any function is defined as

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

If we associate $f(x)$ with $(\partial y/\partial x)_B$ and $f(x)$ with $(\partial y/\partial x)_A$, we see that, in the limit $\Delta x \to 0$, Equation 16.25 becomes

$$\frac{\mu}{T} \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$$

(16.26)

This is the linear wave equation as it applies to waves on a string.

We now show that the sinusoidal wave function (Eq. 16.10) represents a solution of the linear wave equation. If we take the sinusoidal wave function to be of the form $y(x, t) = A \sin(kx - \omega t)$, then the appropriate derivatives are

$$\frac{\partial^2 y}{\partial t^2} = -A \omega^2 \sin(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A \sin(kx - \omega t)$$

Substituting these expressions into Equation 16.26, we obtain

$$-\frac{\mu \omega^2}{T} \sin(kx - \omega t) = -k^2 \sin(kx - \omega t)$$

This equation must be true for all values of the variables $x$ and $t$ in order for the sinusoidal wave function to be a solution of the wave equation. Both sides of the equation depend on $x$ and $t$ through the same function $\sin(kx - \omega t)$. Because this function divides out, we do indeed have an identity, provided that

$$k^2 = \frac{\mu}{T} \omega^2$$

Using the relationship $v = \omega / k$ (Eq. 16.11) in this expression, we see that

$$v^2 = \frac{\omega^2}{k^2} = \frac{T}{\mu}$$

$$v = \sqrt{\frac{T}{\mu}}$$

which is Equation 16.18. This derivation represents another proof of the expression for the wave speed on a taut string.

The linear wave equation (Eq. 16.26) is often written in the form

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

(16.27)

This expression applies in general to various types of traveling waves. For waves on strings, $y$ represents the vertical position of elements of the string. For sound waves, $y$ corresponds to longitudinal position of elements of air from equilibrium or variations in either the pressure or the density of the gas through which the sound waves are propagating. In the case of electromagnetic waves, $y$ corresponds to electric or magnetic field components.

We have shown that the sinusoidal wave function (Eq. 16.10) is one solution of the linear wave equation (Eq. 16.27). Although we do not prove it here, the linear wave equation is satisfied by any wave function having the form $y = f(x \pm vt)$. Furthermore, we have seen that the linear wave equation is a direct consequence of Newton’s second law applied to any element of a string carrying a traveling wave.
A transverse wave is one in which the elements of the medium move in a direction perpendicular to the direction of propagation. An example is a wave on a taut string. A longitudinal wave is one in which the elements of the medium move in a direction parallel to the direction of propagation. Sound waves in fluids are longitudinal.

Any one-dimensional wave traveling with a speed $v$ in the $x$ direction can be represented by a wave function of the form

$$y(x, t) = f(x \pm vt)$$

where the positive sign applies to a wave traveling in the negative $x$ direction and the negative sign applies to a wave traveling in the positive $x$ direction. The shape of the wave at any instant in time (a snapshot of the wave) is obtained by holding $t$ constant.

The wave function for a one-dimensional sinusoidal wave traveling to the right can be expressed as

$$y = A \sin \left( \frac{2\pi}{\lambda} (x - vt) \right) = A \sin (kx - \omega t)$$

where $A$ is the amplitude, $\lambda$ is the wavelength, $k$ is the angular wave number, and $\omega$ is the angular frequency. If $T$ is the period and $f$ the frequency, $v$, $k$, and $\omega$ can be written

$$v = \frac{\lambda}{T} = \lambda f$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

The speed of a wave traveling on a taut string of mass per unit length $\mu$ and tension $T$ is

$$v = \sqrt{\frac{T}{\mu}}$$

A wave is totally or partially reflected when it reaches the end of the medium in which it propagates or when it reaches a boundary where its speed changes discontinuously. If a wave traveling on a string meets a fixed end, the wave is reflected and inverted. If the wave reaches a free end, it is reflected but not inverted.

The power transmitted by a sinusoidal wave on a stretched string is

$$\mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v$$

Wave functions are solutions to a differential equation called the linear wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

### QUESTIONS

1. Why is a pulse on a string considered to be transverse?
2. How would you create a longitudinal wave in a stretched spring? Would it be possible to create a transverse wave in a spring?
3. By what factor would you have to multiply the tension in a stretched string in order to double the wave speed?
4. When traveling on a taut string, does a pulse always invert upon reflection? Explain.
5. Does the vertical speed of a segment of a horizontal taut string, through which a wave is traveling, depend on the wave speed?
6. If you shake one end of a taut rope steadily three times each second, what would be the period of the sinusoidal wave set up in the rope?
7. A vibrating source generates a sinusoidal wave on a string under constant tension. If the power delivered to the
string is doubled, by what factor does the amplitude change? Does the wave speed change under these circumstances?

8. Consider a wave traveling on a taut rope. What is the difference, if any, between the speed of the wave and the speed of a small segment of the rope?

9. If a long rope is hung from a ceiling and waves are sent up the rope from its lower end, they do not ascend with constant speed. Explain.

10. How do transverse waves differ from longitudinal waves?

11. When all the strings on a guitar are stretched to the same tension, will the speed of a wave along the most massive bass string be faster, slower, or the same as the speed of a wave on the lighter strings?

12. If one end of a heavy rope is attached to one end of a light rope, the speed of a wave will change as the wave goes from the heavy rope to the light one. Will it increase or decrease? What happens to the frequency? To the wavelength?

13. If you stretch a rubber hose and pluck it, you can observe a pulse traveling up and down the hose. What happens to the speed of the pulse if you stretch the hose more tightly? What happens to the speed if you fill the hose with water?

14. In a longitudinal wave in a spring, the coils move back and forth in the direction of wave motion. Does the speed of the wave depend on the maximum speed of each coil?

15. Both longitudinal and transverse waves can propagate through a solid. A wave on the surface of a liquid can involve both longitudinal and transverse motion of elements of the medium. On the other hand, a wave propagating through the volume of a fluid must be purely longitudinal, not transverse. Why?

16. In an earthquake both S (transverse) and P (longitudinal) waves propagate from the focus of the earthquake. The focus is in the ground below the epicenter on the surface. The S waves travel through the Earth more slowly than the P waves (at about 5 km/s versus 8 km/s). By detecting the time of arrival of the waves, how can one determine the distance to the focus of the quake? How many detection stations are necessary to locate the focus unambiguously?

17. In mechanics, massless strings are often assumed. Why is this not a good assumption when discussing waves on strings?

### PROBLEMS

**Section 16.1 Propagation of a Disturbance**

1. At \( t = 0 \), a transverse pulse in a wire is described by the function

\[
y = \frac{6}{x^2 + 3}
\]

where \( x \) and \( y \) are in meters. Write the function \( y(x, t) \) that describes this pulse if it is traveling in the positive \( x \) direction with a speed of 4.50 m/s.

2. Ocean waves with a crest-to-crest distance of 10.0 m can be described by the wave function

\[
y(x, t) = (0.800 \text{ m}) \sin[0.628(x - vt)]
\]

where \( v = 1.20 \text{ m/s} \). (a) Sketch \( y(x, t) \) at \( t = 0 \). (b) Sketch \( y(x, t) \) at \( t = 2.00 \text{ s} \). Note that the entire wave form has shifted 2.40 m in the positive \( x \) direction in this time interval.

3. A pulse moving along the \( x \) axis is described by

\[
y(x, t) = 5.00e^{-(x + 5.00t)^2}
\]

where \( x \) is in meters and \( t \) is in seconds. Determine (a) the direction of the wave motion, and (b) the speed of the pulse.

4. Two points \( A \) and \( B \) on the surface of the Earth are at the same longitude and 60.0° apart in latitude. Suppose that an earthquake at point \( A \) creates a P wave that reaches point \( B \) by traveling straight through the body of the Earth at a constant speed of 7.80 km/s. The earthquake also radiates a Rayleigh wave, which travels across the surface of the Earth in an analogous way to a surface wave on water, at 4.50 km/s. (a) Which of these two seismic waves arrives at \( B \) first? (b) What is the time difference between the arrivals of the two waves at \( B \)? Take the radius of the Earth to be 6370 km.

5. S and P waves, simultaneously radiated from the hypocenter of an earthquake, are received at a seismographic station 17.3 s apart. Assume the waves have traveled over the same path at speeds of 4.50 km/s and 7.80 km/s. Find the distance from the seismograph to the hypocenter of the quake.

### Section 16.2 Sinusoidal Waves

6. For a certain transverse wave, the distance between two successive crests is 1.20 m, and eight crests pass a given point along the direction of travel every 12.0 s. Calculate the wave speed.

7. A sinusoidal wave is traveling along a rope. The oscillator that generates the wave completes 40.0 vibrations in 30.0 s. Also, a given maximum travels 425 cm along the rope in 10.0 s. What is the wavelength?

8. When a particular wire is vibrating with a frequency of 4.00 Hz, a transverse wave of wavelength 60.0 cm is produced. Determine the speed of waves along the wire.

9. A wave is described by \( y = (2.00 \text{ cm}) \sin(kx - \omega t) \), where \( k = 2.11 \text{ rad/m}, \omega = 3.62 \text{ rad/s} \). \( x \) is in meters, and \( t \) is in seconds. Determine the amplitude, wavelength, frequency, and speed of the wave.

10. A sinusoidal wave on a string is described by

\[
y = (0.51 \text{ cm}) \sin(kx - \omega t)
\]
where \( k = 3.10 \text{ rad/cm} \) and \( \omega = 9.30 \text{ rad/s} \). How far does a wave crest move in 10.0 s? Does it move in the positive or negative \( x \) direction?

11. Consider further the string shown in Figure 16.10 and treated in Example 16.3. Calculate (a) the maximum transverse speed and (b) the maximum transverse acceleration of a point on the string.

12. Consider the sinusoidal wave of Example 16.2, with the wave function

\[ y = (15.0 \text{ cm}) \cos(0.157x - 50.3t) \]

At a certain instant, let point \( A \) be at the origin and point \( B \) be the first point along the \( x \) axis where the wave is 60.0° out of phase with point \( A \). What is the coordinate of point \( B \)?

13. A sinusoidal wave is described by

\[ y = (0.25 \text{ m}) \sin(0.30x - 40t) \]

where \( x \) and \( y \) are in meters and \( t \) is in seconds. Determine for this wave the (a) amplitude, (b) angular frequency, (c) angular wave number, (d) wavelength, (e) wave speed, and (f) direction of motion.

14. (a) Plot \( y \) versus \( t \) at \( x = 0 \) for a sinusoidal wave of the form \( y = (15.0 \text{ cm}) \cos(0.157x - 50.3t) \), where \( x \) and \( y \) are in centimeters and \( t \) is in seconds. (b) Determine the period of vibration from this plot and compare your result with the value found in Example 16.2.

15. (a) Write the expression for \( y \) as a function of \( x \) and \( t \) for a sinusoidal wave traveling along a rope in the negative \( x \) direction with the following characteristics: \( A = 8.00 \text{ cm} \), \( \lambda = 80.0 \text{ cm} \), \( f = 3.00 \text{ Hz} \), and \( y(0, t) = 0 \) at \( t = 0 \). (b) What If? Write the expression for \( y \) as a function of \( x \) and \( t \) for the wave in part (a) assuming that \( y(x, 0) = 0 \) at the point \( x = 10.0 \text{ cm} \).

16. A sinusoidal wave traveling in the \(-x\) direction (to the left) has an amplitude of 20.0 cm, a wavelength of 35.0 cm, and a frequency of 12.0 Hz. The transverse position of an element of the medium at \( t = 0 \), \( x = 0 \) is \( y = -3.00 \text{ cm} \), and the element has a positive velocity here. (a) Sketch the wave at \( t = 0 \). (b) Find the angular wave number, period, angular frequency, and wave speed of the wave. (c) Write an expression for the wave function \( y(x, t) \).

17. A transverse wave on a string is described by the wave function

\[ y = (0.120 \text{ m}) \sin[(\pi x/8) + 4\pi t] \]

(a) Determine the transverse speed and acceleration at \( t = 0.200 \text{ s} \) for the point on the string located at \( x = 1.60 \text{ m} \). (b) What are the wavelength, period, and speed of propagation of this wave?

18. A transverse sinusoidal wave on a string has a period \( T = 25.0 \text{ ms} \) and travels in the negative \( x \) direction with a speed of 30.0 m/s. At \( t = 0 \), a particle on the string at \( x = 0 \) has a transverse position of 2.00 cm and is traveling downward with a speed of 2.00 m/s. (a) What is the amplitude of the wave? (b) What is the initial phase angle? (c) What is the maximum transverse speed of the string? (d) Write the wave function for the wave.

19. A sinusoidal wave of wavelength 2.00 m and amplitude 0.100 m travels on a string with a speed of 1.00 m/s to the right. Initially, the left end of the string is at the origin. Find (a) the frequency and angular frequency, (b) the angular wave number, and (c) the wave function for this wave. Determine the equation of motion for (d) the left end of the string and (e) the point on the string at \( x = 1.50 \text{ m} \) to the right of the left end. (f) What is the maximum speed of any point on the string?

20. A wave on a string is described by the wave function

\[ y = (0.100 \text{ m}) \sin(0.50x - 200t) \]. (a) Show that a particle in the string at \( x = 2.00 \text{ m} \) executes simple harmonic motion. (b) Determine the frequency of oscillation of this particular point.

Section 16.3 The Speed of Waves on Strings

21. A telephone cord is 4.00 m long. The cord has a mass of 0.200 kg. A transverse pulse is produced by plucking one end of the taut cord. The pulse makes four trips down and back along the cord in 0.800 s. What is the tension in the cord?

22. Transverse waves with a speed of 50.0 m/s are to be produced in a taut string. A 5.00-m length of string with a total mass of 0.060 0 kg is used. What is the required tension?

23. A piano string having a mass per unit length equal to 5.00 \( \times \) \( 10^{-3} \text{ kg/m} \) is under a tension of 1 350 N. Find the speed of a wave traveling on this string.

24. A transverse traveling wave on a taut wire has an amplitude of 0.200 mm and a frequency of 500 Hz. It travels with a speed of 196 m/s. (a) Write an equation in SI units of the form \( y = A \sin(kx - \omega t) \) for this wave. (b) The mass per unit length of this wire is 4.10 g/m. Find the tension in the wire.

25. An astronaut on the Moon wishes to measure the local value of the free-fall acceleration by timing pulses traveling down a wire that has an object of large mass suspended from it. Assume a wire has a mass of 4.00 g and a length of 1.60 m, and that a 3.00-kg object is suspended from it. A pulse requires 36.1 ms to traverse the length of the wire. Calculate \( g_{\text{Moon}} \) from these data. (You may ignore the mass of the wire when calculating the tension in it.)

26. Transverse pulses travel with a speed of 200 m/s along a taut copper wire whose diameter is 1.50 mm. What is the tension in the wire? (The density of copper is 8.92 g/cm³.)

27. Transverse waves travel with a speed of 20.0 m/s in a string under a tension of 6.00 N. What tension is required for a wave speed of 30.0 m/s in the same string?

28. A simple pendulum consists of a ball of mass \( M \) hanging from a uniform string of mass \( m \) and length \( L \) with \( m \ll M \). If the period of oscillations for the pendulum is \( T \), determine the speed of a transverse wave in the string when the pendulum hangs at rest.

29. The elastic limit of the steel forming a piece of wire is equal to 2.70 \( \times \) \( 10^8 \text{ Pa} \). What is the maximum speed at which transverse wave pulses can propagate along this wire without exceeding this stress? (The density of steel is 7.86 \( \times \) \( 10^3 \text{ kg/m}^3 \).)

30. Review problem. A light string with a mass per unit length of 8.00 g/m has its ends tied to two walls separated by a
distance equal to three fourths of the length of the string (Fig. P16.30). An object of mass $m$ is suspended from the center of the string, putting a tension in the string.

(a) Find an expression for the transverse wave speed in the string as a function of the mass of the hanging object.

(b) What should be the mass of the object suspended from the string in order to produce a wave speed of 60.0 m/s?

A taut rope has a mass of 0.180 kg and a length of 3.60 m)

A 3.00-m steel wire and a 2.00-m copper wire, both with 1.00-mm diameters, are connected end to end and stretched to a tension of 150 N. How long does it take to form a transverse wave to travel the entire length of the two wires?

32. Review problem. A light string of mass $m$ and length $L$ has its ends tied to two walls that are separated by the distance $D$. Two objects, each of mass $M$, are suspended from the string as in Figure P16.32. If a wave pulse is sent from point $A$, how long does it take to travel to point $B$?

A sinusoidal wave on a string is described by the equation

$$y = (0.15 \text{ m}) \sin(0.80x - 50t)$$

where $x$ and $y$ are in meters and $t$ is in seconds. If the mass per unit length of this string is 12.0 g/m, determine (a) the speed of the wave, (b) the wavelength, (c) the frequency, and (d) the power transmitted to the wave.

34. A taut rope has a mass of 0.180 kg and a length of 3.60 m.
What power must be supplied to the rope in order to generate sinusoidal waves having an amplitude of 0.100 m and a wavelength of 0.500 m and traveling with a speed of 30.0 m/s?

35. A two-dimensional water wave spreads in circular ripples. Show that the amplitude $A$ at a distance $r$ from the initial disturbance is proportional to $1/\sqrt{r}$. (Suggestion: Consider the energy carried by one outward-moving ripple.)

36. Transverse waves are being generated on a rope under constant tension. By what factor is the required power increased or decreased if (a) the length of the rope is doubled and the angular frequency remains constant, (b) the amplitude is doubled and the angular frequency is halved, (c) both the wavelength and the amplitude are doubled, and (d) both the length of the rope and the wavelength are halved?

37. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string that has a linear mass density of $4.00 \times 10^{-2}$ kg/m. If the source can deliver a maximum power of 300 W and the string is under a tension of 100 N, what is the highest frequency at which the source can operate?

38. It is found that a 6.00-m segment of a long string contains four complete waves and has a mass of 180 g. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley distance of 15.0 cm. (The “peak-to-valley” distance is the vertical distance from the farthest positive position to the farthest negative position.) (a) Write the function that describes this wave traveling in the positive x direction. (b) Determine the power being supplied to the string.

39. A sinusoidal wave on a string is described by the equation

$$y = (0.350 \text{ m}) \sin(10\pi t - 3\pi x + \pi/4)$$

where $x$ is in meters and $t$ is in seconds. (a) What is the average rate at which energy is transmitted along the string if the linear mass density is 75.0 g/m? (b) What is the energy contained in each cycle of the wave?

40. A horizontal string can transmit a maximum power $P_0$ (without breaking) if a wave with amplitude $A$ and angular frequency $\omega$ is traveling along it. In order to increase this maximum power, a student folds the string and uses this “double string” as a medium. Determine the maximum power that can be transmitted along the “double string,” assuming that the tension is constant.

41. In a region far from the epicenter of an earthquake, a seismic wave can be modeled as transporting energy in a single direction without absorption, just as a string wave does. Suppose the seismic wave moves from granite into mudfill with similar density but with a much lower bulk modulus. Assume the speed of the wave gradually drops by a factor of 25.0, with negligible reflection of the wave. Will the amplitude of the ground shaking increase or decrease? By
what factor? This phenomenon led to the collapse of part of the Nimitz Freeway in Oakland, California, during the Loma Prieta earthquake of 1989.

Section 16.6 The Linear Wave Equation

43. (a) Evaluate $A$ in the scalar equality $(7 + 3)4 = A$. (b) Evaluate $A$, $B$, and $C$ in the vector equality $7.00\hat{i} + 3.00\hat{k} = A\hat{i} + B\hat{j} + C\hat{k}$. Explain how you arrive at the answers to convince a student who thinks that you cannot solve a single equation for three different unknowns. (c) What If? The functional equality or identity

$$A + B \cos(Cx + Dt + E) = (7.00 \text{ mm}) \cos(3x + 4t + 2)$$

is true for all values of the variables $x$ and $t$, which are measured in meters and in seconds, respectively. Evaluate the constants $A$, $B$, $C$, $D$, and $E$. Explain how you arrive at the answers.

44. Show that the wave function $y = e^{j(x-vt)}$ is a solution of the linear wave equation (Eq. 16.27), where $\delta$ is a constant.

45. Show that the wave function $y = \ln\{b(x-vt)\}$ is a solution to Equation 16.27, where $b$ is a constant.

46. (a) Show that the function $y(x, t) = x^2 + v^2t^2$ is a solution to the wave equation. (b) Show that the function in part (a) can be written as $f(x+vt) + g(x-vt)$, and determine the functional forms for $f$ and $g$. (c) What If? Repeat parts (a) and (b) for the function $y(x, t) = \sin(x)\cos(vt)$.

Additional Problems

47. “The wave” is a particular type of pulse that can propagate through a large crowd gathered at a sports arena to watch a soccer or American football match (Figure P16.47). The elements of the medium are the spectators, with zero position corresponding to their being seated and maximum position corresponding to their standing and raising their arms. When a large fraction of the spectators participate in the wave motion, a somewhat stable pulse shape can develop. The wave speed depends on people’s reaction time, which is typically on the order of 0.1 s. Estimate the order of magnitude, in minutes, of the time required for such a pulse to make one circuit around a large sports stadium. State the quantities you measure or estimate and their values.

48. A traveling wave propagates according to the expression $y = (4.0 \text{ cm}) \sin(2.0x - 3.0t)$, where $x$ is in centimeters and $t$ is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, (d) the period, and (e) the direction of travel of the wave.

49. The wave function for a traveling wave on a taut string is (in SI units)

$$y(x, t) = (0.350 \text{ m}) \sin(10\pi t - 3\pi x + \pi/4)$$

(a) What are the speed and direction of travel of the wave? (b) What is the vertical position of an element of the string at $t = 0, x = 0.100 \text{ m}$? (c) What are the wavelength and frequency of the wave? (d) What is the maximum magnitude of the transverse speed of the string?

50. A transverse wave on a string is described by the equation

$$y(x, t) = (0.350 \text{ m}) \sin\left((1.25 \text{ rad/m})x + (99.6 \text{ rad/s})t\right)$$

Consider the element of the string at $x = 0$. (a) What is the time interval between the first two instants when this element has a position of $y = 0.175 \text{ m}$? (b) What distance does the wave travel during this time interval?

51. Motion picture film is projected at 24.0 frames per second. Each frame is a photograph 19.0 mm high. At what constant speed does the film pass into the projector?

52. Review problem. A block of mass $M$, supported by a string, rests on an incline making an angle $\theta$ with the horizontal (Fig. P16.52). The length of the string is $L$, and its mass is $m \ll M$. Derive an expression for the time interval required for a transverse wave to travel from one end of the string to the other.

![Figure P16.47](Gregg Adams/Getty Images)

53. Review problem. A 2.00-kg block hangs from a rubber cord, being supported so that the cord is not stretched. The unstretched length of the cord is 0.500 m, and its mass is 5.00 g. The “spring constant” for the cord is 100 N/m. The block is released and stops at the lowest
54. Review problem. A block of mass $M$ hangs from a rubber cord. The block is supported so that the cord is not stretched. The unstretched length of the cord is $L_0$ and its mass is $m$, much less than $M$. The “spring constant” for the cord is $k$. The block is released and stops at the lowest point. (a) Determine the tension in the string when the block is at this lowest point. (b) What is the length of the cord in this “stretched” position? (c) Find the speed of a transverse wave in the cord if the block is held in this lowest position.

55. (a) Determine the speed of transverse waves on a string under a tension of 80.0 N if the string has a length of 2.00 m and a mass of 5.00 g. (b) Calculate the power required to generate these waves if they have a wavelength of 16.0 cm and an amplitude of 4.00 cm.

56. A sinusoidal wave in a rope is described by the wave function

$$y = (0.20 \text{ m}) \sin(0.75\pi x + 18\pi t)$$

where $x$ and $y$ are in meters and $t$ is in seconds. The rope has a linear mass density of 0.250 kg/m. If the tension in the rope is provided by an arrangement like the one illustrated in Figure 16.12, what is the value of the suspended mass?

57. A block of mass 0.450 kg is attached to one end of a cord of mass 0.003 20 kg; the other end of the cord is attached to a fixed point. The block rotates with constant angular speed in a circle on a horizontal frictionless table. Through what angle does the block rotate in the time that a transverse wave takes to travel along the string from the center of the circle to the block?

58. A wire of density $\rho$ is tapered so that its cross-sectional area varies with $x$ according to

$$A = (1.0 \times 10^{-3} x + 0.010) \text{ cm}^2$$

(a) If the wire is subject to a tension $T$, derive a relationship for the speed of a wave as a function of position. (b) What If? If the wire is aluminum and is subject to a tension of 24.0 N, determine the speed at the origin and at $x = 10.0$ m.

59. A rope of total mass $m$ and length $L$ is suspended vertically. Show that a transverse pulse travels the length of the rope in a time interval $\Delta t = 2\sqrt{L/g}$. (Suggestion: First find an expression for the wave speed at any point a distance $x$ from the lower end by considering the tension in the rope as resulting from the weight of the segment below that point.)

60. If an object of mass $M$ is suspended from the bottom of the rope in Problem 59, (a) show that the time interval for a transverse pulse to travel the length of the rope is

$$\Delta t = 2\sqrt{\frac{L}{mg}} \left(\sqrt{M + m} - \sqrt{M}\right)$$

What If? (b) Show that this reduces to the result of Problem 59 when $M = 0$. (c) Show that for $m \ll M$, the expression in part (a) reduces to

$$\Delta t = \sqrt{\frac{mL}{Mg}}$$

It is stated in Problem 59 that a pulse travels from the bottom to the top of a hanging rope of length $L$ in a time interval $\Delta t = 2\sqrt{L/g}$. Use this result to answer the following questions. (It is not necessary to set up any new integrations.) (a) How long does it take for a pulse to travel halfway up the rope? Give your answer as a fraction of the quantity $2L/g$. (b) A pulse starts traveling up the rope. How far has it traveled after a time interval $\sqrt{L/g}$?

62. Determine the speed and direction of propagation of each of the following sinusoidal waves, assuming that $x$ and $y$ are measured in meters and $t$ in seconds.

(a) $y = 0.60 \cos(3.0x - 15t + 2)$
(b) $y = 0.40 \cos(3.0x + 15t - 2)$
(c) $y = 1.2 \sin(15t + 2.0x)$
(d) $y = 0.20 \sin(12t - (x/2) + \pi)$

An aluminum wire is clamped at each end under zero tension at room temperature. The tension in the wire is increased by reducing the temperature, which results in a decrease in the wire’s equilibrium length. What strain $\Delta L/L$ results in a transverse wave speed of 100 m/s? Take the cross-sectional area of the wire to be $5.00 \times 10^{-6}$ m², the density to be $2.70 \times 10^3$ kg/m³, and Young’s modulus to be $7.00 \times 10^10$ N/m².

64. If a loop of chain is spun at high speed, it can roll along the ground like a circular hoop without slipping or collapsing. Consider a chain of uniform linear mass density $\mu$ whose center of mass travels to the right at a high speed $v_0$. (a) Determine the tension in the chain in terms of $\mu$ and $v_0$. (b) If the loop rolls over a bump, the resulting deformation of the chain causes two transverse pulses to propagate along the chain, one moving clockwise and one moving counterclockwise. What is the speed of the pulses traveling along the chain? (c) Through what angle does each pulse travel during the time it takes the loop to make one revolution?

65. (a) Show that the speed of longitudinal waves along a spring of force constant $k$ is $v = \sqrt{kL/\mu}$, where $L$ is the unstretched length of the spring and $\mu$ is the mass per unit length. (b) A spring with a mass of 0.400 kg has an unstretched length of 2.00 m and a force constant of 100 N/m. Using the result you obtained in (a), determine the speed of longitudinal waves along this spring.

66. A string of length $L$ consists of two sections. The left half has mass per unit length $\mu = \mu_0/2$, while the right has a mass per unit length $\mu' = 3\mu = 3\mu_0/2$. Tension in the string is $T_0$. Notice from the data given that this string has the same total mass as a uniform string of length $L$ and mass per unit length $\mu_0$. (a) Find the speeds $v$ and $v'$ at which transverse pulses travel in the two sections. Express the speeds in terms of $T_0$ and $\mu_0$, and also as multiples of the speed $v_0 = (T_0/\mu_0)^{1/2}$. (b) Find the time interval required for a pulse to travel from one end
of the string to the other. Give your result as a multiple of $\Delta t_0 = L/v_0$.

67. A pulse traveling along a string of linear mass density $\mu$ is described by the wave function

$$y = [A_0 e^{-bx}] \sin(kx - \omega t)$$

where the factor in brackets before the sine function is said to be the amplitude. (a) What is the power $P(x)$ carried by this wave at a point $x$? (b) What is the power carried by this wave at the origin? (c) Compute the ratio $P(x)/P(0)$.

68. An earthquake on the ocean floor in the Gulf of Alaska produces a tsunami (sometimes incorrectly called a “tidal wave”) that reaches Hilo, Hawaii, 4450 km away, in a time interval of 9 h 30 min. Tsunamis have enormous wavelengths (100 to 200 km), and the propagation speed for these waves is $v \approx \sqrt{gd}$, where $d$ is the average depth of the water. From the information given, find the average wave speed and the average ocean depth between Alaska and Hawaii. (This method was used in 1856 to estimate the average depth of the Pacific Ocean long before soundings were made to give a direct determination.)

69. A string on a musical instrument is held under tension $T$ and extends from the point $x = 0$ to the point $x = L$. The string is overwound with wire in such a way that its mass per unit length $\mu(x)$ increases uniformly from $\mu_0$ at $x = 0$ to $\mu_L$ at $x = L$. (a) Find an expression for $\mu(x)$ as a function of $x$ over the range $0 \leq x \leq L$. (b) Show that the time interval required for a transverse pulse to travel the length of the string is given by

$$\Delta t = \frac{2L(\mu_L + \mu_0 + \sqrt{\mu_L\mu_0})}{3\sqrt{T(\sqrt{\mu_L} + \sqrt{\mu_0})}}$$

**Answers to Quick Quizzes**

16.1 (b). It is longitudinal because the disturbance (the shift of position of the people) is parallel to the direction in which the wave travels.

16.2 (a). It is transverse because the people stand up and sit down (vertical motion), whereas the wave moves either to the left or to the right.

16.3 (c). The wave speed is determined by the medium, so it is unaffected by changing the frequency.

16.4 (b). Because the wave speed remains the same, the result of doubling the frequency is that the wavelength is half as large.

16.5 (d). The amplitude of a wave is unrelated to the wave speed, so we cannot determine the new amplitude without further information.

16.6 (c). With a larger amplitude, an element of the string has more energy associated with its simple harmonic motion, so the element passes through the equilibrium position with a higher maximum transverse speed.

16.7 Only answers (f) and (h) are correct. (a) and (b) affect the transverse speed of a particle of the string, but not the wave speed along the string. (c) and (d) change the amplitude. (e) and (g) increase the time interval by decreasing the wave speed.

16.8 (d). Doubling the amplitude of the wave causes the power to be larger by a factor of 4. In (a), halving the linear mass density of the string causes the power to change by a factor of 0.71—the rate decreases. In (b), doubling the wavelength of the wave halves the frequency and causes the power to change by a factor of 0.25—the rate decreases. In (c), doubling the tension in the string changes the wave speed and causes the power to change by a factor of 1.4—not as large as in part (d).
Human ears have evolved to detect sound waves and interpret them as music or speech. Some animals, such as this young bat-eared fox, have ears adapted for the detection of very weak sounds. (Getty Images)
Sound waves are the most common example of longitudinal waves. They travel through any material medium with a speed that depends on the properties of the medium. As the waves travel through air, the elements of air vibrate to produce changes in density and pressure along the direction of motion of the wave. If the source of the sound waves vibrates sinusoidally, the pressure variations are also sinusoidal. The mathematical description of sinusoidal sound waves is very similar to that of sinusoidal string waves, which were discussed in the previous chapter.

Sound waves are divided into three categories that cover different frequency ranges. (1) Audible waves lie within the range of sensitivity of the human ear. They can be generated in a variety of ways, such as by musical instruments, human voices, or loudspeakers. (2) Infrasonic waves have frequencies below the audible range. Elephants can use infrasonic waves to communicate with each other, even when separated by many kilometers. (3) Ultrasonic waves have frequencies above the audible range. You may have used a "silent" whistle to retrieve your dog. The ultrasonic sound it emits is easily heard by dogs, although humans cannot detect it at all. Ultrasonic waves are also used in medical imaging.

We begin this chapter by discussing the speed of sound waves and then wave intensity, which is a function of wave amplitude. We then provide an alternative description of the intensity of sound waves that compresses the wide range of intensities to which the ear is sensitive into a smaller range for convenience. We investigate the effects of the motion of sources and/or listeners on the frequency of a sound. Finally, we explore digital reproduction of sound, focusing in particular on sound systems used in modern motion pictures.

### 17.1 Speed of Sound Waves

Let us describe pictorially the motion of a one-dimensional longitudinal pulse moving through a long tube containing a compressible gas (Fig. 17.1). A piston at the left end can be moved to the right to compress the gas and create the pulse. Before the piston is moved, the gas is undisturbed and of uniform density, as represented by the uniformly shaded region in Figure 17.1a. When the piston is suddenly pushed to the right (Fig. 17.1b), the gas just in front of it is compressed (as represented by the more heavily shaded region); the pressure and density in this region are now higher than they were before the piston moved. When the piston comes to rest (Fig. 17.1c), the compressed region of the gas continues to move to the right, corresponding to a longitudinal pulse traveling through the tube with speed \( v \). Note that the piston speed does not equal \( v \). Furthermore, the compressed region does not "stay with" the piston as the piston moves, because the speed of the wave is usually greater than the speed of the piston.

The speed of sound waves in a medium depends on the compressibility and density of the medium. If the medium is a liquid or a gas and has a bulk modulus \( B \) (see Figure 17.1) Motion of a longitudinal pulse through a compressible gas. The compression (darker region) is produced by the moving piston.
Table 17.1

<table>
<thead>
<tr>
<th>Medium</th>
<th>(\nu) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gases</strong></td>
<td></td>
</tr>
<tr>
<td>Hydrogen (0°C)</td>
<td>1 286</td>
</tr>
<tr>
<td>Helium (0°C)</td>
<td>972</td>
</tr>
<tr>
<td>Air (20°C)</td>
<td>343</td>
</tr>
<tr>
<td>Air (0°C)</td>
<td>331</td>
</tr>
<tr>
<td>Oxygen (0°C)</td>
<td>317</td>
</tr>
<tr>
<td><strong>Liquids at 25°C</strong></td>
<td></td>
</tr>
<tr>
<td>Glycerol</td>
<td>1 904</td>
</tr>
<tr>
<td>Seawater</td>
<td>1 533</td>
</tr>
<tr>
<td>Water</td>
<td>1 493</td>
</tr>
<tr>
<td>Mercury</td>
<td>1 450</td>
</tr>
<tr>
<td>Kerosene</td>
<td>1 324</td>
</tr>
<tr>
<td>Methyl alcohol</td>
<td>1 143</td>
</tr>
<tr>
<td>Carbon tetrachloride</td>
<td>926</td>
</tr>
<tr>
<td><strong>Solids</strong></td>
<td></td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>5 640</td>
</tr>
<tr>
<td>Iron</td>
<td>5 950</td>
</tr>
<tr>
<td>Aluminum</td>
<td>6 420</td>
</tr>
<tr>
<td>Brass</td>
<td>4 700</td>
</tr>
<tr>
<td>Copper</td>
<td>5 010</td>
</tr>
<tr>
<td>Gold</td>
<td>3 240</td>
</tr>
<tr>
<td>Lucite</td>
<td>2 680</td>
</tr>
<tr>
<td>Lead</td>
<td>1 960</td>
</tr>
<tr>
<td>Rubber</td>
<td>1 600</td>
</tr>
</tbody>
</table>

* Values given are for propagation of longitudinal waves in bulk media. Speeds for longitudinal waves in thin rods are smaller, and speeds of transverse waves in bulk are smaller yet.

Section 12.4) and density \(\rho\), the speed of sound waves in that medium is

\[
\nu = \sqrt{\frac{B}{\rho}}
\]  

(17.1)

It is interesting to compare this expression with Equation 16.18 for the speed of transverse waves on a string, \(\nu = \sqrt{\frac{T}{\mu}}\). In both cases, the wave speed depends on an elastic property of the medium—bulk modulus \(B\) or string tension \(T\)—and on an inertial property of the medium—\(\rho\) or \(\mu\). In fact, the speed of all mechanical waves follows an expression of the general form

\[
\nu = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}
\]

For longitudinal sound waves in a solid rod of material, for example, the speed of sound depends on Young’s modulus \(Y\) and the density \(\rho\). Table 17.1 provides the speed of sound in several different materials.

The speed of sound also depends on the temperature of the medium. For sound traveling through air, the relationship between wave speed and medium temperature is

\[
\nu = (331 \text{ m/s})\sqrt{1 + \frac{T_C}{273\text{°C}}}
\]

where 331 m/s is the speed of sound in air at 0°C, and \(T_C\) is the air temperature in degrees Celsius. Using this equation, one finds that at 20°C the speed of sound in air is approximately 343 m/s.

This information provides a convenient way to estimate the distance to a thunderstorm. You count the number of seconds between seeing the flash of lightning and hearing the thunder. Dividing this time by 3 gives the approximate distance to the lightning in kilometers, because 343 m/s is approximately \(\frac{1}{3}\) km/s. Dividing the time in seconds by 5 gives the approximate distance to the lightning in miles, because the speed of sound in ft/s (1 125 ft/s) is approximately \(\frac{1}{3}\) mi/s.

Quick Quiz 17.1 The speed of sound in air is a function of (a) wavelength (b) frequency (c) temperature (d) amplitude.

Example 17.1 Speed of Sound in a Liquid

(A) Find the speed of sound in water, which has a bulk modulus of \(2.1 \times 10^9\) N/m² at a temperature of 0°C and a density of \(1.00 \times 10^3\) kg/m³.

**Solution** Using Equation 17.1, we find that

\[
\nu_{\text{water}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.1 \times 10^9 \text{ N/m}^2}{1.00 \times 10^3 \text{ kg/m}^3}} = 1.4 \text{ km/s}
\]

In general, sound waves travel more slowly in liquids than in solids because liquids are more compressible than solids. Note that the speed of sound in water is lower at 0°C than at 25°C (Table 17.1).

(B) Dolphins use sound waves to locate food. Experiments have shown that a dolphin can detect a 7.5-cm target 110 m away, even in murky water. For a bit of “dinner” at that distance, how much time passes between the moment the dolphin emits a sound pulse and the moment the dolphin hears its reflection and thereby detects the distant target?

**Solution** The total distance covered by the sound wave as it travels from dolphin to target and back is \(2 \times 110\) m = 220 m. From Equation 2.2, we have, for 25°C water

\[
\Delta t = \frac{\Delta x}{\nu_x} = \frac{220 \text{ m}}{1 533 \text{ m/s}} = 0.14 \text{ s}
\]

At the Interactive Worked Example link at http://www.pse6.com, you can compare the speed of sound through the various media found in Table 17.1.
17.2 Periodic Sound Waves

This section will help you better comprehend the nature of sound waves. An important fact for understanding how our ears work is that pressure variations control what we hear.

One can produce a one-dimensional periodic sound wave in a long, narrow tube containing a gas by means of an oscillating piston at one end, as shown in Figure 17.2. The darker parts of the colored areas in this figure represent regions where the gas is compressed and thus the density and pressure are above their equilibrium values. A compressed region is formed whenever the piston is pushed into the tube. This compressed region, called a compression, moves through the tube as a pulse, continuously compressing the region just in front of itself. When the piston is pulled back, the gas in front of it expands, and the pressure and density in this region fall below their equilibrium values (represented by the lighter parts of the colored areas in Fig. 17.2). These low-pressure regions, called rarefactions, also propagate along the tube, following the compressions. Both regions move with a speed equal to the speed of sound in the medium.

As the piston oscillates sinusoidally, regions of compression and rarefaction are continuously set up. The distance between two successive compressions (or two successive rarefactions) equals the wavelength \( \lambda \). As these regions travel through the tube, any small element of the medium moves with simple harmonic motion parallel to the direction of the wave. If \( s(x, t) \) is the position of a small element relative to its equilibrium position,\(^1\) we can express this harmonic position function as

\[
s(x, t) = s_{\text{max}} \cos(kx - \omega t) \tag{17.2}
\]

where \( s_{\text{max}} \) is the maximum position of the element relative to equilibrium. This is often called the displacement amplitude of the wave. The parameter \( k \) is the wave number and \( \omega \) is the angular frequency of the piston. Note that the displacement of the element is along \( x \), in the direction of propagation of the sound wave, which means we are describing a longitudinal wave.

The variation in the gas pressure \( \Delta P \) measured from the equilibrium value is also periodic. For the position function in Equation 17.2, \( \Delta P \) is given by

\[
\Delta P = \Delta P_{\text{max}} \sin(kx - \omega t) \tag{17.3}
\]

where the pressure amplitude \( \Delta P_{\text{max}} \)—which is the maximum change in pressure from the equilibrium value—is given by

\[
\Delta P_{\text{max}} = \rho v_0 \omega s_{\text{max}} \tag{17.4}
\]

Thus, we see that a sound wave may be considered as either a displacement wave or a pressure wave. A comparison of Equations 17.2 and 17.3 shows that the pressure wave is \( 90^\circ \) out of phase with the displacement wave. Graphs of these functions are shown in Figure 17.3. Note that the pressure variation is a maximum when the displacement from equilibrium is zero, and the displacement from equilibrium is a maximum when the pressure variation is zero.

**Quick Quiz 17.2** If you blow across the top of an empty soft-drink bottle, a pulse of sound travels down through the air in the bottle. At the moment the pulse reaches the bottom of the bottle, the correct descriptions of the displacement of elements of air from their equilibrium positions and the pressure of the air at this point are (a) the displacement and pressure are both at a maximum (b) the displacement and pressure are both at a minimum (c) the displacement is zero and the pressure is a maximum (d) the displacement is zero and the pressure is a minimum.

\(^1\) We use \( s(x, t) \) here instead of \( y(x, t) \) because the displacement of elements of the medium is not perpendicular to the \( x \) direction.
Derivation of Equation 17.3

Consider a thin disk-shaped element of gas whose circular cross section is parallel to the piston in Figure 17.2. This element will undergo changes in position, pressure, and density as a sound wave propagates through the gas. From the definition of bulk modulus (see Eq. 12.8), the pressure variation in the gas is

$$\Delta P = -B \frac{\Delta V}{V_i}$$

The element has a thickness $\Delta x$ in the horizontal direction and a cross-sectional area $A$, so its volume is $V_i = A \Delta x$. The change in volume $\Delta V$ accompanying the pressure change is equal to $A \Delta s$, where $\Delta s$ is the difference between the value of $s$ at $x + \Delta x$ and the value of $s$ at $x$. Hence, we can express $\Delta P$ as

$$\Delta P = -B \frac{\Delta V}{V_i} = -B \frac{A \Delta s}{A \Delta x} = -B \frac{\Delta s}{\Delta x}$$

As $\Delta x$ approaches zero, the ratio $\Delta s/\Delta x$ becomes $\partial s/\partial x$. (The partial derivative indicates that we are interested in the variation of $s$ with position at a fixed time.) Therefore,

$$\Delta P = -B \frac{\partial s}{\partial x}$$

If the position function is the simple sinusoidal function given by Equation 17.2, we find that

$$\Delta P = -B \frac{\partial}{\partial x} \left[ s_{\text{max}} \cos(kx - \omega t) \right] = B s_{\text{max}} k \sin(kx - \omega t)$$

Because the bulk modulus is given by $B = \rho v^2$ (see Eq. 17.1), the pressure variation reduces to

$$\Delta P = \rho v^2 s_{\text{max}} k \sin(kx - \omega t)$$

From Equation 16.11, we can write $k = \omega / v$; hence, $\Delta P$ can be expressed as

$$\Delta P = \rho v \omega s_{\text{max}} \sin(kx - \omega t)$$

Because the sine function has a maximum value of 1, we see that the maximum value of the pressure variation is $\Delta P_{\text{max}} = \rho v \omega s_{\text{max}}$ (see Eq. 17.4), and we arrive at Equation 17.3:

$$\Delta P = \Delta P_{\text{max}} \sin(kx - \omega t)$$

17.3 Intensity of Periodic Sound Waves

In the preceding chapter, we showed that a wave traveling on a taut string transports energy. The same concept applies to sound waves. Consider an element of air of mass $\Delta m$ and width $\Delta x$ in front of a piston oscillating with a frequency $\omega$, as shown in Figure 17.4.
The piston transmits energy to this element of air in the tube, and the energy is propagated away from the piston by the sound wave. To evaluate the rate of energy transfer for the sound wave, we shall evaluate the kinetic energy of this element of air, which is undergoing simple harmonic motion. We shall follow a procedure similar to that in Section 16.5, in which we evaluated the rate of energy transfer for a wave on a string.

As the sound wave propagates away from the piston, the position of any element of air in front of the piston is given by Equation 17.2. That in Section 16.5, in which we evaluated the rate of energy transfer for a wave on a string.

Imagine that we take a “snapshot” of the wave at \( t = 0 \). The kinetic energy of a given element of air at this time is

\[
\Delta K = \frac{1}{2} \Delta m(v)^2 = \frac{1}{2} \Delta m(-\omega_{\text{max}} \sin kx)^2
\]

where \( A \) is the cross-sectional area of the element and \( A \Delta x \) is its volume. Now, as in Section 16.5, we integrate this expression over a full wavelength to find the total kinetic energy in one wavelength. Letting the element of air shrink to infinitesimal thickness, so that \( \Delta x \to dx \), we have

\[
K_A = \int dK = \int_0^\lambda \frac{1}{2} \rho A (\omega_{\text{max}})^2 \sin^2 kx \, dx = \frac{1}{2} \rho A (\omega_{\text{max}})^2 \int_0^\lambda \sin^2 kx \, dx
\]

As in the case of the string wave in Section 16.5, the total potential energy for one wavelength has the same value as the total kinetic energy; thus, the total mechanical energy for one wavelength is

\[
E_A = K_A + U_A = \frac{1}{2} \rho A (\omega_{\text{max}})^2 \lambda
\]

As the sound wave moves through the air, this amount of energy passes by a given point during one period of oscillation. Hence, the rate of energy transfer is

\[
\mathcal{P} = \frac{\Delta E}{\Delta t} = \frac{E_A}{T} = \frac{\frac{1}{2} \rho A (\omega_{\text{max}})^2 \lambda}{T} = \frac{1}{2} \rho A v (\omega_{\text{max}})^2 \left( \frac{\lambda}{T} \right) = \frac{1}{2} \rho A v (\omega_{\text{max}})^2
\]

where \( v \) is the speed of sound in air.

We define the **intensity** \( I \) of a wave, or the power per unit area, to be the rate at which the energy being transported by the wave transfers through a unit area \( A \) perpendicular to the direction of travel of the wave:

\[
I = \frac{\mathcal{P}}{A}
\]

In the present case, therefore, the intensity is

\[
I = \frac{\mathcal{P}}{A} = \frac{1}{2} \rho v (\omega_{\text{max}})^2
\]

Thus, we see that the intensity of a periodic sound wave is proportional to the square of the displacement amplitude and to the square of the angular frequency (as in the case of a periodic string wave). This can also be written in terms of the pressure.
amplitude $\Delta P_{\text{max}}$; in this case, we use Equation 17.4 to obtain

$$I = \frac{\Delta P_{\text{max}}^2}{2\rho v}$$  \hspace{1cm} (17.6)

Now consider a point source emitting sound waves equally in all directions. From everyday experience, we know that the intensity of sound decreases as we move farther from the source. We identify an imaginary sphere of radius $r$ centered on the source. When a source emits sound equally in all directions, we describe the result as a **spherical wave**. The average power $P_{\text{av}}$ emitted by the source must be distributed uniformly over this spherical surface of area $4\pi r^2$. Hence, the wave intensity at a distance $r$ from the source is

$$I = \frac{P_{\text{av}}}{A} = \frac{P_{\text{av}}}{4\pi r^2}$$  \hspace{1cm} (17.7)

This inverse-square law, which is reminiscent of the behavior of gravity in Chapter 13, states that the intensity decreases in proportion to the square of the distance from the source.

**Quick Quiz 17.3** An *ear trumpet* is a cone-shaped shell, like a megaphone, that was used before hearing aids were developed to help persons who were hard of hearing. The small end of the cone was held in the ear, and the large end was aimed toward the source of sound as in Figure 17.5. The ear trumpet increases the intensity of sound because (a) it increases the speed of sound (b) it reflects sound back toward the source (c) it gathers sound that would normally miss the ear and concentrates it into a smaller area (d) it increases the density of the air.

![Figure 17.5](Quick Quiz 17.3) An ear trumpet, used before hearing aids to make sounds intense enough for people who were hard of hearing. You can simulate the effect of an ear trumpet by cupping your hands behind your ears.
Quick Quiz 17.4  A vibrating guitar string makes very little sound if it is not
mounted on the guitar. But if this vibrating string is attached to the guitar body, so that the
body of the guitar vibrates, the sound is higher in intensity. This is because (a) the power
of the vibration is spread out over a larger area (b) the energy leaves the guitar at a higher
rate (c) the speed of sound is higher in the material of the guitar body (d) none of these.

Example 17.2  Hearing Limits

The faintest sounds the human ear can detect at a
frequency of 1 000 Hz correspond to an intensity of
about 1.00 \times 10^{-12} \text{ W/m}^2—the so-called threshold of hear-
ing. The loudest sounds the ear can tolerate at this
frequency correspond to an intensity of about 1.00 \text{ W/m}^2—
the threshold of pain. Determine the pressure amplitude
and displacement amplitude associated with these two
limits.

Solution

First, consider the faintest sounds. Using Equation 17.6 and taking \( v = 343 \text{ m/s} \) as the speed of sound
waves in air and \( p = 1.20 \text{ kg/m}^3 \) as the density of air, we ob-
tain

\[
\Delta p_{\text{max}} = \sqrt{2 \rho v I} \\
= \sqrt{2(1.20 \text{ kg/m}^3)(343 \text{ m/s})(1.00 \times 10^{-12} \text{ W/m}^2)} \\
= 2.87 \times 10^{-5} \text{ N/m}^2
\]

Because atmospheric pressure is about 10^5 \text{ N/m}^2, this result
tells us that the ear is sensitive to pressure fluctuations as small as 3 parts in 10^{10}.

We can calculate the corresponding displacement amplitude by using Equation 17.4, recalling that \( \omega = 2\pi f \) (see Eqs. 16.3 and 16.9):

\[
I_{\text{max}} = \frac{\Delta p_{\text{max}}}{\rho \omega} = \frac{2.87 \times 10^{-5} \text{ N/m}^2}{(1.20 \text{ kg/m}^3)(343 \text{ m/s})(2\pi \times 1000 \text{ Hz})} \\
= 1.11 \times 10^{-11} \text{ m}
\]

This is a remarkably small number! If we compare this result
for \( I_{\text{max}} \) with the size of an atom (about 10^{-10} \text{ m}), we see
that the ear is an extremely sensitive detector of sound waves.

In a similar manner, one finds that the loudest sounds
the human ear can tolerate correspond to a pressure ampli-
tude of 28.7 \text{ N/m}^2 and a displacement amplitude equal to
1.11 \times 10^{-5} \text{ m}.

Example 17.3  Intensity Variations of a Point Source

A point source emits sound waves with an average power
output of 80.0 W.

(A) Find the intensity 3.00 m from the source.

Solution

A point source emits energy in the form of spherical
waves. Using Equation 17.7, we have

\[
I = \frac{\rho_{\text{av}} v}{4\pi r^2} = \frac{80.0 \text{ W}}{4\pi(3.00 \text{ m})^2} = 0.707 \text{ W/m}^2
\]

an intensity that is close to the threshold of pain.

(B) Find the distance at which the intensity of the sound is
1.00 \times 10^{-8} \text{ W/m}^2.

Solution

Using this value for \( I \) in Equation 17.7 and solving
for \( r \), we obtain

\[
r = \sqrt{\frac{\rho_{\text{av}} v}{4\pi I}} = \sqrt{\frac{80.0 \text{ W}}{4\pi(1.00 \times 10^{-8} \text{ W/m}^2)}} \\
= 2.52 \times 10^4 \text{ m}
\]

which equals about 16 miles!

Sound Level in Decibels

Example 17.2 illustrates the wide range of intensities the human ear can detect. Be-
cause this range is so wide, it is convenient to use a logarithmic scale, where the sound
level \( \beta \) (Greek beta) is defined by the equation

\[
\beta = 10 \log \left( \frac{I}{I_0} \right) \tag{17.8} \text{ Sound level in decibels}
\]

The constant \( I_0 \) is the reference intensity, taken to be at the threshold of hearing
\( (I_0 = 1.00 \times 10^{-12} \text{ W/m}^2) \), and \( I \) is the intensity in watts per square meter to which
the sound level \( \beta \) corresponds, where \( \beta \) is measured^2 in decibels (dB). On this scale,

^2 The unit bel is named after the inventor of the telephone, Alexander Graham Bell (1847–1922).
The prefix deci- is the SI prefix that stands for 10\(^{-1}\).
Chapter 17 • Sound Waves

Example 17.4 Sound Levels

Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each machine at the location of the worker is \( I = 2.0 \times 10^{-7} \text{ W/m}^2 \). Find the sound level heard by the worker when one machine is operating and when both machines are operating.

### Solution

(A) When one machine is operating, the sound level at the location of the worker with one machine operating is calculated from Equation 17.8:

\[
\beta_1 = 10 \log \left( \frac{2.0 \times 10^{-7} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2} \right) = 10 \log(2.0 \times 10^5) = 53 \text{ dB}
\]

(B) When both machines are operating, the intensity is doubled to \( 4.0 \times 10^{-7} \text{ W/m}^2 \); therefore, the sound level now is

\[
\beta_2 = 10 \log \left( \frac{4.0 \times 10^{-7} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2} \right) = 10 \log(4.0 \times 10^5) = 56 \text{ dB}
\]

From these results, we see that when the intensity is doubled, the sound level increases by only 3 dB.

**What If?** Loudness is a psychological response to a sound and depends on both the intensity and the frequency of the sound. As a rule of thumb, a doubling in loudness is approximately associated with an increase in sound level of 10 dB. (Note that this rule of thumb is relatively inaccurate at very low or very high frequencies.) If the loudness of the

<table>
<thead>
<tr>
<th>Source of Sound</th>
<th>( \beta ) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearby jet airplane</td>
<td>150</td>
</tr>
<tr>
<td>Jackhammer; machine gun</td>
<td>130</td>
</tr>
<tr>
<td>Siren; rock concert</td>
<td>120</td>
</tr>
<tr>
<td>Subway; power mower</td>
<td>100</td>
</tr>
<tr>
<td>Busy traffic</td>
<td>80</td>
</tr>
<tr>
<td>Vacuum cleaner</td>
<td>70</td>
</tr>
<tr>
<td>Normal conversation</td>
<td>50</td>
</tr>
<tr>
<td>Mosquito buzzing</td>
<td>40</td>
</tr>
<tr>
<td>Whisper</td>
<td>30</td>
</tr>
<tr>
<td>Rustling leaves</td>
<td>10</td>
</tr>
<tr>
<td>Threshold of hearing</td>
<td>0</td>
</tr>
</tbody>
</table>

**Quick Quiz 17.5** A violin plays a melody line and is then joined by a second violin, playing at the same intensity as the first violin, in a repeat of the same melody. With both violins playing, what physical parameter has doubled compared to the situation with only one violin playing? (a) wavelength (b) frequency (c) intensity (d) sound level in dB (e) none of these.

**Quick Quiz 17.6** Increasing the intensity of a sound by a factor of 100 causes the sound level to increase by (a) 100 dB (b) 20 dB (c) 10 dB (d) 2 dB.
machines in this example is to be doubled, how many machines must be running?

**Answer** Using the rule of thumb, a doubling of loudness corresponds to a sound level increase of 10 dB. Thus,

$$\beta_2 - \beta_1 = 10 \log \left( \frac{I_2}{I_0} \right) - 10 \log \left( \frac{I_1}{I_0} \right) = 10 \log \left( \frac{I_2}{I_1} \right),$$

Thus, ten machines must be operating to double the loudness.

**Loudness and Frequency**

The discussion of sound level in decibels relates to a *physical* measurement of the strength of a sound. Let us now consider how we describe the *psychological* “measurement” of the strength of a sound.

Of course, we don’t have meters in our bodies that can read out numerical values of our reactions to stimuli. We have to “calibrate” our reactions somehow by comparing different sounds to a reference sound. However, this is not easy to accomplish. For example, earlier we mentioned that the threshold intensity is $10^{-12}$ W/m$^2$, corresponding to an intensity level of 0 dB. In reality, this value is the threshold only for a sound of frequency 1 000 Hz, which is a standard reference frequency in acoustics. If we perform an experiment to measure the threshold intensity at other frequencies, we find a distinct variation of this threshold as a function of frequency. For example, at 100 Hz, a sound must have an intensity level of about 30 dB in order to be just barely audible! Unfortunately, there is no simple relationship between physical measurements and psychological “measurements.”

The 100-Hz, 30-dB sound is psychologically “equal” to the 1 000-Hz, 0-dB sound (both are just barely audible) but they are not physically equal ($30 \text{ dB} \neq 0 \text{ dB}$).

By using test subjects, the human response to sound has been studied, and the results are shown in Figure 17.6 (the white area), along with the approximate frequency and sound-level ranges of other sound sources. The lower curve of the white area corresponds to the threshold of hearing. Its variation with frequency is clear from this diagram. Note that humans are sensitive to frequencies ranging from about 20 Hz to about 20 000 Hz. The upper bound of the white area is the threshold of pain. Here the

![Figure 17.6](From R. L. Reese, University Physics, Pacific Grove, Brooks/Cole, 2000.)

**Figure 17.6** Approximate frequency and sound level ranges of various sources and that of normal human hearing, shown by the white area.
boundary of the white area is straight, because the psychological response is relatively independent of frequency at this high sound level.

The most dramatic change with frequency is in the lower left region of the white area, for low frequencies and low intensity levels. Our ears are particularly insensitive in this region. If you are listening to your stereo and the bass (low frequencies) and treble (high frequencies) sound balanced at a high volume, try turning the volume down and listening again. You will probably notice that the bass seems weak, which is due to the insensitivity of the ear to low frequencies at low sound levels, as shown in Figure 17.6.

17.4 The Doppler Effect

Perhaps you have noticed how the sound of a vehicle’s horn changes as the vehicle moves past you. The frequency of the sound you hear as the vehicle approaches you is higher than the frequency you hear as it moves away from you. This is one example of the Doppler effect.

To see what causes this apparent frequency change, imagine you are in a boat that is lying at anchor on a gentle sea where the waves have a period of \( T = 3.0 \text{ s} \). This means that every 3.0 s a crest hits your boat. Figure 17.7a shows this situation, with the water waves moving toward the left. If you set your watch to \( t = 0 \) just as one crest hits, the watch reads 3.0 s when the next crest hits, 6.0 s when the third crest hits, and so on. From these observations you conclude that the wave frequency is \( f = 1/T = 1/(3.0 \text{ s}) = 0.33 \text{ Hz} \). Now suppose you start your motor and head directly into the oncoming waves, as in Figure 17.7b. Again you set your watch to \( t = 0 \) as a crest hits the front of your boat. Now, however, because you are moving toward the next wave crest as it moves toward you, it hits you less than 3.0 s after the first hit. In other words, the period you observe is shorter than the 3.0-s period you observed when you were stationary. Because \( f = 1/T \), you observe a higher wave frequency than when you were at rest.

If you turn around and move in the same direction as the waves (see Fig. 17.7c), you observe the opposite effect. You set your watch to \( t = 0 \) as a crest hits the back of the boat. Because you are now moving away from the next crest, more than 3.0 s has elapsed on your watch by the time that crest catches you. Thus, you observe a lower frequency than when you were at rest.

These effects occur because the relative speed between your boat and the waves depends on the direction of travel and on the speed of your boat. When you are moving toward the right in Figure 17.7b, this relative speed is higher than that of the wave speed, which leads to the observation of an increased frequency. When you turn around and move to the left, the relative speed is lower, as is the observed frequency of the water waves.

Let us now examine an analogous situation with sound waves, in which the water waves become sound waves, the water becomes the air, and the person on the boat becomes an observer listening to the sound. In this case, an observer \( O \) is moving and a sound source \( S \) is stationary. For simplicity, we assume that the air is also stationary and that the observer moves directly toward the source (Fig. 17.8). The observer moves with a speed \( v_O \) toward a stationary point source \( (v_S = 0) \), where stationary means at rest with respect to the medium, air.

If a point source emits sound waves and the medium is uniform, the waves move at the same speed in all directions radially away from the source; this is a spherical wave, as was mentioned in Section 17.3. It is useful to represent these waves with a series of circular arcs concentric with the source, as in Figure 17.8. Each arc represents a surface over which the phase of the wave is constant. For example, the surface could pass through the crests of all waves. We call such a surface of constant phase a wave front. The distance between adjacent wave fronts equals the wavelength \( \lambda \). In Figure 17.8, the

---

5 Named after the Austrian physicist Christian Johann Doppler (1803–1853), who in 1842 predicted the effect for both sound waves and light waves.
circles are the intersections of these three-dimensional wave fronts with the two-dimensional paper.

We take the frequency of the source in Figure 17.8 to be \( f \), the wavelength to be \( \lambda \), and the speed of sound to be \( v \). If the observer were also stationary, he or she would detect wave fronts at a rate \( f \). (That is, when \( v_O = 0 \) and \( v_S = 0 \), the observed frequency equals the source frequency.) When the observer moves toward the source, the speed of the waves relative to the observer is \( v' = v + v_O \), as in the case of the boat, but the

**Active Figure 17.8** An observer \( O \) (the cyclist) moves with a speed \( v_O \) toward a stationary point source \( S \), the horn of a parked truck. The observer hears a frequency \( f' \) that is greater than the source frequency.
wavelength $\lambda$ is unchanged. Hence, using Equation 16.12, $v = \lambda f$, we can say that the frequency $f'$ heard by the observer is *increased* and is given by

$$f' = \frac{v'}{\lambda} = \frac{v + v_O}{\lambda}$$

Because $\lambda = \frac{v}{f}$, we can express $f'$ as

$$f' = \left(\frac{v + v_O}{v}\right) f \quad \text{(observer moving toward source)} \quad (17.9)$$

If the observer is moving away from the source, the speed of the wave relative to the observer is $v' = v - v_O$. The frequency heard by the observer in this case is *decreased* and is given by

$$f' = \left(\frac{v - v_O}{v}\right) f \quad \text{(observer moving away from source)} \quad (17.10)$$

In general, whenever an observer moves with a speed $v_O$ relative to a stationary source, the frequency heard by the observer is given by Equation 17.9, with a sign convention: a positive value is substituted for $v_O$ when the observer moves toward the source and a negative value is substituted when the observer moves away from the source.

Now consider the situation in which the source is in motion and the observer is at rest. If the source moves directly toward observer A in Figure 17.9a, the wave fronts heard by the observer are closer together than they would be if the source were not moving. As a result, the wavelength $\lambda'$ measured by observer A is shorter than the wavelength $\lambda$ of the source. During each vibration, which lasts for a time interval $T$ (the period), the source moves a distance $v_S T = \frac{v_S}{f}$ and the wavelength is shortened by this amount. Therefore, the observed wavelength $\lambda'$ is

$$\lambda' = \lambda - \Delta\lambda = \lambda - \frac{v_S}{f}$$

Because $\lambda = \frac{v}{f}$, the frequency $f'$ heard by observer A is

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - (v_S/f)} = \frac{v}{(v/f) - (v_S/f)}$$

**(a)** A source $S$ moving with a speed $v_S$ toward a stationary observer A and away from a stationary observer B. Observer A hears an increased frequency, and observer B hears a decreased frequency. (b) The Doppler effect in water, observed in a ripple tank. A point source is moving to the right with speed $v_S$. Letters shown in the photo refer to Quick Quiz 17.7.
That is, the observed frequency is increased whenever the source is moving toward the observer.

When the source moves away from a stationary observer, as is the case for observer B in Figure 17.9a, the observer measures a wavelength $\lambda'$ that is greater than $\lambda$ and hears a decreased frequency:

$$f' = \left(\frac{v}{v + v_s}\right) f \quad \text{(source moving away from observer)} \quad (17.12)$$

We can express the general relationship for the observed frequency when a source is moving and an observer is at rest as Equation 17.11, with the same sign convention applied to $v_S$ as was applied to $v_O$: a positive value is substituted for $v_S$ when the source moves toward the observer and a negative value is substituted when the source moves away from the observer.

Finally, we find the following general relationship for the observed frequency:

$$f' = \left(\frac{v + v_O}{v - v_S}\right) f \quad (17.13)$$

In this expression, the signs for the values substituted for $v_O$ and $v_S$ depend on the direction of the velocity. A positive value is used for motion of the observer or the source toward the other, and a negative sign for motion of one away from the other.

A convenient rule concerning signs for you to remember when working with all Doppler-effect problems is as follows:

The word toward is associated with an increase in observed frequency. The words away from are associated with a decrease in observed frequency.

Although the Doppler effect is most typically experienced with sound waves, it is a phenomenon that is common to all waves. For example, the relative motion of source and observer produces a frequency shift in light waves. The Doppler effect is used in police radar systems to measure the speeds of motor vehicles. Likewise, astronomers use the effect to determine the speeds of stars, galaxies, and other celestial objects relative to the Earth.

**Quick Quiz 17.7** Consider detectors of water waves at three locations A, B, and C in Figure 17.9b. Which of the following statements is true? (a) The wave speed is highest at location A. (b) The wave speed is highest at location C. (c) The detected wavelength is largest at location B. (c) The detected wavelength is largest at location C. (e) The detected frequency is highest at location C. (f) The detected frequency is highest at location A.

**Quick Quiz 17.8** You stand on a platform at a train station and listen to a train approaching the station at a constant velocity. While the train approaches, but before it arrives, you hear (a) the intensity and the frequency of the sound both increasing (b) the intensity and the frequency of the sound both decreasing (c) the intensity increasing and the frequency decreasing (d) the intensity decreasing and the frequency increasing (e) the intensity increasing and the frequency remaining the same (f) the intensity decreasing and the frequency remaining the same.

**PITFALL PREVENTION**

17.1 Doppler Effect Does Not Depend on Distance

Many people think that the Doppler effect depends on the distance between the source and the observer. While the intensity of a sound varies as the distance changes, the apparent frequency depends only on the relative speed of source and observer. As you listen to an approaching source, you will detect increasing intensity but constant frequency. As the source passes, you will hear the frequency suddenly drop to a new constant value and the intensity begin to decrease.
Example 17.5 The Broken Clock Radio

Your clock radio awakens you with a steady and irritating sound of frequency 600 Hz. One morning, it malfunctions and cannot be turned off. In frustration, you drop the clock radio out of your fourth-story dorm window, 15.0 m from the ground. Assume the speed of sound is 343 m/s.

(A) As you listen to the falling clock radio, what frequency do you hear just before you hear the radio striking the ground?

(B) At what rate does the frequency that you hear change with time just before you hear the radio striking the ground?

Solution

(A) In conceptualizing the problem, note that the speed of the radio increases as it falls. Thus, it is a source of sound moving away from you with an increasing speed. We categorize this problem as one in which we must combine our understanding of falling objects with that of the frequency shift due to the Doppler effect. To analyze the problem, we identify the clock radio as a moving source of sound for which the Doppler-shifted frequency is given by

\[ f' = \left( \frac{v}{v - v_s} \right) f \]

The speed of the source of sound is given by Equation 2.9 for a falling object:

\[ v_s = v_{yi} + a_y t = 0 - gt = -gt \]

Thus, the Doppler-shifted frequency of the falling clock radio is

\[ f' = \left( \frac{v}{v - (-gt)} \right) f = \left( \frac{v}{v + gt} \right) f \]

The time at which the radio strikes the ground is found from Equation 2.12:

\[ y_f = y_i + v_{yi} t - \frac{1}{2}gt^2 \]

\[ -15.0 \text{ m} = 0 + 0 - \frac{1}{2}(9.80 \text{ m/s}^2)t^2 \]

\[ t = 1.75 \text{ s} \]

Thus, the Doppler-shifted frequency just as the radio strikes the ground is

\[ f' = \left( \frac{v}{v + gt} \right) f \]

\[ = \left( \frac{343 \text{ m/s}}{343 \text{ m/s} + (9.80 \text{ m/s}^2)(1.75 \text{ s})} \right)(600 \text{ Hz}) \]

\[ = 571 \text{ Hz} \]

(B) The rate at which the frequency changes is found by differentiating Equation (1) with respect to \( t \):

\[ \frac{df'}{dt} = \frac{d}{dt} \left( \frac{vf}{v + gt} \right) = \frac{-v v_y t f}{(v + gt)^2} \]

\[ = \frac{-v (v)(9.80 \text{ m/s}^2)}{[343 \text{ m/s} + (9.80 \text{ m/s}^2)(1.75 \text{ s})]^2} (600 \text{ Hz}) \]

\[ = -15.5 \text{ Hz/s} \]

To finalize this problem, consider the following What If?

What If? Suppose you live on the eighth floor instead of the fourth floor. If you repeat the radio-dropping activity, does the frequency shift in part (A) and the rate of change of frequency in part (B) of this example double?

Answer The doubled height does not give a time at which the radio lands that is twice the time found in part (A). From Equation 2.12:

\[ y_f = y_i + v_{yi} t - \frac{1}{2}gt^2 \]

\[ -30.0 \text{ m} = 0 + 0 - \frac{1}{2}(9.80 \text{ m/s}^2)t^2 \]

\[ t = 2.47 \text{ s} \]

The new frequency heard just before you hear the radio strike the ground is

\[ f' = \left( \frac{v}{v + gt} \right) f \]

\[ = \left( \frac{343 \text{ m/s}}{343 \text{ m/s} + (9.80 \text{ m/s}^2)(2.47 \text{ s})} \right)(600 \text{ Hz}) \]

\[ = 560 \text{ Hz} \]

The frequency shift heard on the fourth floor is 600 Hz - 571 Hz = 29 Hz, while the frequency shift heard from the eighth floor is 600 Hz - 560 Hz = 40 Hz, which is not twice as large.

The new rate of change of frequency is

\[ \frac{df'}{dt} = \frac{-v v_y t f}{(v + gt)^2} \]

\[ = \frac{-v (v)(9.80 \text{ m/s}^2)}{[343 \text{ m/s} + (9.80 \text{ m/s}^2)(2.47 \text{ s})]^2} (600 \text{ Hz}) \]

\[ = -15.0 \text{ Hz/s} \]

Note that this value is actually smaller in magnitude than the previous value of -15.5 Hz/s!

Example 17.6 Doppler Submarines

A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1 400 Hz. The speed of sound in the water is 1 533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly toward one another. The second submarine is moving at 9.00 m/s.

(A) What frequency is detected by an observer riding on sub B as the subs approach each other?

(B) The subs barely miss each other and pass. What frequency is detected by an observer riding on sub B as the subs recede from each other?
### Solution

**A** We use Equation 17.13 to find the Doppler-shifted frequency. As the two submarines approach each other, the observer in sub B hears the frequency

\[
f' = \left( \frac{v + v_Q}{v - v_S} \right) f
\]

\[
= \left( \frac{1.533 \text{ m/s} + (+9.00 \text{ m/s})}{1.533 \text{ m/s} - (+8.00 \text{ m/s})} \right) (1400 \text{ Hz}) = 1416 \text{ Hz}
\]

**B** As the two submarines recede from each other, the observer in sub B hears the frequency

\[
f' = \left( \frac{v + v_Q}{v - v_S} \right) f
\]

\[
= \left( \frac{1.533 \text{ m/s} + (-9.00 \text{ m/s})}{1.533 \text{ m/s} - (-8.00 \text{ m/s})} \right) (1400 \text{ Hz}) = 1385 \text{ Hz}
\]

### What If?
While the subs are approaching each other, some of the sound from sub A will reflect from sub B and return to sub A. If this sound were to be detected by an observer on sub A, what is its frequency?

**Answer** The sound of apparent frequency 1416 Hz found in part (A) will be reflected from a moving source (sub B) and then detected by a moving observer (sub A). Thus, the frequency detected by sub A is

\[
f'' = \left( \frac{v + v_Q}{v - v_S} \right) f'
\]

\[
= \left( \frac{1.533 \text{ m/s} + (+8.00 \text{ m/s})}{1.533 \text{ m/s} - (+9.00 \text{ m/s})} \right) (1416 \text{ Hz}) = 1432 \text{ Hz}
\]

This technique is used by police officers to measure the speed of a moving car. Microwaves are emitted from the police car and reflected by the moving car. By detecting the Doppler-shifted frequency of the reflected microwaves, the police officer can determine the speed of the moving car.

---

**Shock Waves**

Now consider what happens when the speed \(v_S\) of a source exceeds the wave speed \(v\). This situation is depicted graphically in Figure 17.10a. The circles represent spherical wave fronts emitted by the source at various times during its motion. At \(t = 0\), the source is at \(S_0\), and at a later time \(t\), the source is at \(S_t\). At the time \(t\), the wave front

![Figure 17.10](image-url)  
**Figure 17.10** (a) A representation of a shock wave produced when a source moves from \(S_0\) to \(S_t\) with a speed \(v_S\), which is greater than the wave speed \(v\) in the medium. The envelope of the wave fronts forms a cone whose apex half-angle is given by \(\sin \theta = v/v_S\). (b) A stroboscopic photograph of a bullet moving at supersonic speed through the hot air above a candle. Note the shock wave in the vicinity of the bullet.
centered at $S_0$ reaches a radius of $vt$. In this same time interval, the source travels a distance $v_t$ to $S_n$. At the instant the source is at $S_n$, waves are just beginning to be generated at this location, and hence the wave front has zero radius at this point. The tangent line drawn from $S_n$ to the wave front centered on $S_0$ is tangent to all other wave fronts generated at intermediate times. Thus, we see that the envelope of these wave fronts is a cone whose apex half-angle $\theta$ (the "Mach angle") is given by

$$\sin \theta = \frac{vt}{v_S} = \frac{v}{v_S}$$

The ratio $v_S/v$ is referred to as the Mach number, and the conical wave front produced when $v_S > v$ (supersonic speeds) is known as a shock wave. An interesting analogy to shock waves is the V-shaped wave fronts produced by a boat (the bow wave) when the boat’s speed exceeds the speed of the surface-water waves (Fig. 17.11).

Jet airplanes traveling at supersonic speeds produce shock waves, which are responsible for the loud “sonic boom” one hears. The shock wave carries a great deal of energy concentrated on the surface of the cone, with correspondingly great pressure variations. Such shock waves are unpleasant to hear and can cause damage to buildings when aircraft fly supersonically at low altitudes. In fact, an airplane flying at supersonic speeds produces a double boom because two shock waves are formed, one from the nose of the plane and one from the tail. People near the path of the space shuttle as it glides toward its landing point often report hearing what sounds like two very closely spaced cracks of thunder.

Quick Quiz 17.9 An airplane flying with a constant velocity moves from a cold air mass into a warm air mass. Does the Mach number (a) increase (b) decrease (c) stay the same?

17.5 Digital Sound Recording

The first sound recording device, the phonograph, was invented by Thomas Edison in the nineteenth century. Sound waves were recorded in early phonographs by encoding the sound waveforms as variations in the depth of a continuous groove cut in tin foil wrapped around a cylinder. During playback, as a needle followed along the groove of the rotating cylinder, the needle was pushed back and forth according to the sound
waves encoded on the record. The needle was attached to a diaphragm and a horn (Fig. 17.12), which made the sound loud enough to be heard.

As the development of the phonograph continued, sound was recorded on cardboard cylinders coated with wax. During the last decade of the nineteenth century and the first half of the twentieth century, sound was recorded on disks made of shellac and clay. In 1948, the plastic phonograph disk was introduced and dominated the recording industry market until the advent of compact discs in the 1980s.

There are a number of problems with phonograph records. As the needle follows along the groove of the rotating phonograph record, the needle is pushed back and forth according to the sound waves encoded on the record. By Newton’s third law, the needle also pushes on the plastic. As a result, the recording quality diminishes with each playing as small pieces of plastic break off and the record wears away.

Another problem occurs at high frequencies. The wavelength of the sound on the record is so small that natural bumps and graininess in the plastic create signals as loud as the sound signal, resulting in noise. The noise is especially noticeable during quiet passages in which high frequencies are being played. This is handled electronically by a process known as pre-emphasis. In this process, the high frequencies are recorded with more intensity than they actually have, which increases the amplitude of the vibrations and overshadows the sources of noise. Then, an equalization circuit in the playback system is used to reduce the intensity of the high-frequency sounds, which also reduces the intensity of the noise.

**Example 17.7  Wavelengths on a Phonograph Record**

Consider a 10 000-Hz sound recorded on a phonograph record which rotates at 33 1/3 rev/min. How far apart are the crests of the wave for this sound on the record?

(A) at the outer edge of the record, 6.0 inches from the center?

(B) at the inner edge, 1.0 inch from the center?

**Solution**

(A) The linear speed $v$ of a point at the outer edge of the record is $2\pi r/T$ where $T$ is the period of the rotation and $r$
is the distance from the center. We first find \( T \):

\[
T = \frac{1}{f} = \frac{1}{33.33 \text{ rev/min}} = 0.030 \text{ min} \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 1.8 \text{ s}
\]

Now, the linear speed at the outer edge is

\[
v = \frac{2\pi r}{T} = \frac{2\pi(6.0 \text{ in.})}{1.8 \text{ s}} = 21 \text{ in./s} \left( \frac{2.54 \text{ cm}}{1 \text{ in.}} \right)
\]

\[
= 53 \text{ cm/s}
\]

Thus, the wave on the record is moving past the needle at this speed. The wavelength is

\[
\lambda = \frac{v}{f} = \frac{53 \text{ cm/s}}{10 000 \text{ Hz}} = 5.3 \times 10^{-5} \text{ m}
\]

\[
= 53 \mu\text{m}
\]

(B) The linear speed at the inner edge is

\[
v = \frac{2\pi r}{T} = \frac{2\pi(1.0 \text{ in.})}{1.8 \text{ s}} = 3.5 \text{ in.} / \text{s} \left( \frac{2.54 \text{ cm}}{1 \text{ in.}} \right)
\]

\[
= 8.9 \text{ cm/s}
\]

The wavelength is

\[
\lambda = \frac{v}{f} = \frac{8.9 \text{ cm/s}}{10 000 \text{ Hz}} = 8.9 \times 10^{-6} \text{ m}
\]

\[
= 8.9 \mu\text{m}
\]

Thus, the problem with noise interfering with the recorded sound is more severe at the inner edge of the disk than at the outer edge.

**Digital Recording**

In digital recording, information is converted to binary code (ones and zeroes), similar to the dots and dashes of Morse code. First, the waveform of the sound is sampled, typically at the rate of 44 100 times per second. Figure 17.13 illustrates this process. The sampling frequency is much higher than the upper range of hearing, about 20 000 Hz, so all frequencies of sound are sampled at this rate. During each sampling, the pressure of the wave is measured and converted to a voltage. Thus, there are 44 100 numbers associated with each second of the sound being sampled.

These measurements are then converted to binary numbers, which are numbers expressed using base 2 rather than base 10. Table 17.3 shows some sample binary numbers. Generally, voltage measurements are recorded in 16-bit “words,” where each bit is a one or a zero. Thus, the number of different voltage levels that can be assigned codes is \(2^{16} = 65 536\). The number of bits in one second of sound is \(16 \times 44 100 = 705 600\). It is these strings of ones and zeroes, in 16-bit words, that are recorded on the surface of a compact disc.

Figure 17.14 shows a magnification of the surface of a compact disc. There are two types of areas that are detected by the laser playback system—lands and pits. The lands are untouched regions of the disc surface that are highly reflective. The pits, which are areas burned into the surface, scatter light rather than reflecting it back to the detection system. The playback system samples the reflected light 705 600 times per second. When the laser moves from a pit to a flat or from a flat to a pit, the reflected light changes during the sampling and the bit is recorded as a one. If there is no change during the sampling, the bit is recorded as a zero.

**Figure 17.13** Sound is digitized by electronically sampling the sound waveform at periodic intervals. During each time interval between the blue lines, a number is recorded for the average voltage during the interval. The sampling rate shown here is much slower than the actual sampling rate of 44 100 samples per second.
The binary numbers read from the CD are converted back to voltages, and the waveform is reconstructed, as shown in Figure 17.15. Because the sampling rate is so high—44 100 voltage readings each second—the fact that the waveform is constructed from step-wise discrete voltages is not evident in the sound.

The advantage of digital recording is in the high fidelity of the sound. With analog recording, any small imperfection in the record surface or the recording equipment can cause a distortion of the waveform. If all peaks of a maximum in a waveform are clipped off so as to be only 90% as high, for example, this will have a major effect on the spectrum of the sound in an analog recording. With digital recording, however, it takes a major imperfection to turn a one into a zero. If an imperfection causes the magnitude of a one to be 90% of the original value, it still registers as a one, and there is no distortion. Another advantage of digital recording is that the information is extracted optically, so that there is no mechanical wear on the disc.

**Table 17.3**

<table>
<thead>
<tr>
<th>Number in Base 10</th>
<th>Number in Binary</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0000000000000001</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0000000000000010</td>
<td>2 + 0</td>
</tr>
<tr>
<td>3</td>
<td>0000000000000011</td>
<td>2 + 1</td>
</tr>
<tr>
<td>10</td>
<td>0000000000001010</td>
<td>8 + 0 + 2 + 0</td>
</tr>
<tr>
<td>37</td>
<td>0000000000010010</td>
<td>32 + 0 + 0 + 4 + 0 + 1</td>
</tr>
<tr>
<td>275</td>
<td>00000010010011</td>
<td>256 + 0 + 0 + 0 + 16 + 0 + 0 + 2 + 1</td>
</tr>
</tbody>
</table>

The binary numbers read from the CD are converted back to voltages, and the waveform is reconstructed, as shown in Figure 17.15. Because the sampling rate is so high—44 100 voltage readings each second—the fact that the waveform is constructed from step-wise discrete voltages is not evident in the sound.

The advantage of digital recording is in the high fidelity of the sound. With analog recording, any small imperfection in the record surface or the recording equipment can cause a distortion of the waveform. If all peaks of a maximum in a waveform are clipped off so as to be only 90% as high, for example, this will have a major effect on the spectrum of the sound in an analog recording. With digital recording, however, it takes a major imperfection to turn a one into a zero. If an imperfection causes the magnitude of a one to be 90% of the original value, it still registers as a one, and there is no distortion. Another advantage of digital recording is that the information is extracted optically, so that there is no mechanical wear on the disc.
Figure 17.15 The reconstruction of the sound wave sampled in Figure 17.13. Notice that the reconstruction is step-wise, rather than the continuous waveform in Figure 17.13.

**Example 17.8 How Big Are the Pits?**

In Example 10.2, we mentioned that the speed with which the CD surface passes the laser is 1.3 m/s. What is the average length of the audio track on a CD associated with each bit of the audio information?

**Solution** In one second, a 1.3-m length of audio track passes by the laser. This length includes 705 600 bits of audio information. Thus, the average length per bit is

\[
\frac{1.3 \text{ m}}{705 \text{ 600 bits}} = 1.8 \times 10^{-6} \text{ m/bit}
\]

The average length per bit of total information on the CD is smaller than this because there is additional information on the disc besides the audio information. This information includes error correction codes, song numbers, timing codes, etc. As a result, the shortest length per bit is actually about 0.8 μm.

**Example 17.9 What’s the Number?**

Consider the photograph of the compact disc surface in Figure 17.14. Audio data undergoes complicated processing in order to reduce a variety of errors in reading the data. Thus, an audio “word” is not laid out linearly on the disc. Suppose that data has been read from the disc, the error encoding has been removed, and the resulting audio word is

1 0 1 1 1 0 1 1 1 0 1 1 0 1 1

What is the decimal number represented by this 16-bit word?

**Solution** We convert each of these bits to a power of 2 and add the results:

\[
\begin{align*}
1 \times 2^{15} &= 32 \, 768 & 1 \times 2^9 &= 512 & 1 \times 2^3 &= 8 \\
0 \times 2^{14} &= 0 & 1 \times 2^8 &= 256 & 0 \times 2^2 &= 0 \\
1 \times 2^{13} &= 8 \, 192 & 1 \times 2^7 &= 128 & 1 \times 2^1 &= 2 \\
1 \times 2^{12} &= 4 \, 096 & 0 \times 2^6 &= 0 & 1 \times 2^0 &= 1 \\
1 \times 2^{11} &= 2 \, 048 & 1 \times 2^5 &= 32 & \\
0 \times 2^{10} &= 0 & 1 \times 2^4 &= 16 & \text{sum} &= 48 \, 059
\end{align*}
\]

This number is converted by the CD player into a voltage, representing one of the 44 100 values that will be used to build one second of the electronic waveform that represents the recorded sound.

### 17.6 Motion Picture Sound

Another interesting application of digital sound is the soundtrack in a motion picture. Early twentieth-century movies recorded sound on phonograph records, which were synchronized with the action on the screen. Beginning with early newsreel films, the *variable-area optical soundtrack* process was introduced, in which sound was recorded on an optical track on the film. The width of the transparent portion of the track varied according to the sound wave that was recorded. A photocell detecting light passing through the track converted the varying light intensity to a sound wave. As with phonograph recording, there are a number of difficulties with this recording system. For example, dirt or fingerprints on the film cause fluctuations in intensity and loss of fidelity.

Digital recording on film first appeared with *Dick Tracy* (1990), using the Cinema Digital Sound (CDS) system. This system suffered from lack of an analog backup system in case of equipment failure and is no longer used in the film industry. It did, however, introduce the use of 5.1 channels of sound—Left, Center, Right, Right Surround, Left Surround, and Low Frequency Effects (LFE). The LFE channel, which is the “0.1
channel” of 5.1, carries very low frequencies for dramatic sound from explosions, earthquakes, and the like.

Current motion pictures are produced with three systems of digital sound recording:

Dolby Digital; In this format, 5.1 channels of digital sound are optically stored between the sprocket holes of the film. There is an analog optical backup in case the digital system fails. The first film to use this technique was *Batman Returns* (1992).

DTS (Digital Theater Sound); 5.1 channels of sound are stored on a separate CD-ROM which is synchronized to the film print by time codes on the film. There is an analog optical backup in case the digital system fails. The first film to use this technique was *Jurassic Park* (1993).

SDDS (Sony Dynamic Digital Sound); Eight full channels of digital sound are optically stored outside the sprocket holes on both sides of film. There is an analog optical backup in case the digital system fails. The first film to use this technique was *Last Action Hero* (1993). The existence of information on both sides of the tape is a system of redundancy—in case one side is damaged, the system will still operate. SDDS employs a full-spectrum LFE channel and two additional channels (left center and right center behind the screen). In Figure 17.16, showing a section of SDDS film, both the analog optical soundtrack and the dual digital soundtracks can be seen.

![Figure 17.16](image-url) The layout of information on motion picture film using the SDDS digital sound system.
Sound waves are longitudinal and travel through a compressible medium with a speed that depends on the elastic and inertial properties of that medium. The speed of sound in a liquid or gas having a bulk modulus $B$ and density $\rho$ is

$$v = \sqrt{\frac{B}{\rho}} \quad (17.1)$$

For sinusoidal sound waves, the variation in the position of an element of the medium is given by

$$s(x, t) = s_{\text{max}} \cos(kx - \omega t) \quad (17.2)$$

and the variation in pressure from the equilibrium value is

$$\Delta P = \Delta P_{\text{max}} \sin(kx - \omega t) \quad (17.3)$$

where $\Delta P_{\text{max}}$ is the pressure amplitude. The pressure wave is $90^\circ$ out of phase with the displacement wave. The relationship between $s_{\text{max}}$ and $\Delta P_{\text{max}}$ is given by

$$\Delta P_{\text{max}} = \rho v s_{\text{max}} \quad (17.4)$$

The intensity of a periodic sound wave, which is the power per unit area, is

$$I = \frac{\mathcal{P}}{A} = \frac{\Delta P_{\text{max}}^2}{2\rho v} \quad (17.5, 17.6)$$

The sound level of a sound wave, in decibels, is given by

$$\beta = 10 \log \left( \frac{I}{I_0} \right) \quad (17.8)$$

The constant $I_0$ is a reference intensity, usually taken to be at the threshold of hearing ($1.00 \times 10^{-12}$ W/m$^2$), and $I$ is the intensity of the sound wave in watts per square meter.

The change in frequency heard by an observer whenever there is relative motion between a source of sound waves and the observer is called the Doppler effect. The observed frequency is

$$f' = \left( \frac{v + v_0}{v - v_S} \right) f \quad (17.13)$$

In this expression, the signs for the values substituted for $v_0$ and $v_S$ depend on the direction of the velocity. A positive value for the velocity of the observer or source is substituted if the velocity of one is toward the other, while a negative value represents a velocity of one away from the other.

In digital recording of sound, the sound waveform is sampled 44 100 times per second. The pressure of the wave for each sampling is measured and converted to a binary number. In playback, these binary numbers are read and used to build the original waveform.

### QUESTIONS

1. Why are sound waves characterized as longitudinal?
2. If an alarm clock is placed in a good vacuum and then activated, no sound is heard. Explain.
3. A sonic ranger is a device that determines the distance to an object by sending out an ultrasonic sound pulse and measuring how long it takes for the wave to return after it reflects from the object. Typically these devices cannot reliably detect an object that is less than half a meter from the sensor. Why is that?
4. A friend sitting in her car far down the road waves to you and beeps her horn at the same time. How far away must she be for you to calculate the speed of sound to two significant figures by measuring the time it takes for the sound to reach you?
5. If the wavelength of sound is reduced by a factor of 2, what happens to its frequency? Its speed?

6. By listening to a band or orchestra, how can you determine that the speed of sound is the same for all frequencies?

7. In Example 17.3 we found that a point source with a power output of 80 W produces sound with an intensity of $1.00 \times 10^{-8}$ W/m$^2$, which corresponds to 40 dB, at a distance of about 16 miles. Why do you suppose you cannot normally hear a rock concert that is going on 16 miles away? (See Table 17.2.)

8. If the distance from a point source is tripled, by what factor does the intensity decrease?

9. The Tunguska Event. On June 30, 1908, a meteor burned up and exploded in the atmosphere above the Tunguska River valley in Siberia. It knocked down trees over thousands of square kilometers and started a forest fire, but apparently caused no human casualties. A witness sitting on his doorstep outside the zone of falling trees recalled events in the following sequence: He saw a moving light in the sky, brighter than the sun and descending at a low angle to the horizon. He felt his face become warm. He felt the ground shake. An invisible agent picked him up and immediately dropped him about a meter farther away from where the light had been. He heard a very loud protracted rumbling. Suggest an explanation for these observations and for the order in which they happened.

10. Explain how the Doppler effect with microwaves is used to determine the speed of an automobile.

11. Explain what happens to the frequency of the echo of your car horn as you move in a vehicle toward the wall of a canyon. What happens to the frequency as you move away from the wall?

12. Of the following sounds, which is most likely to have a sound level of 60 dB: a rock concert, the turning of a page in this textbook, normal conversation, or a cheering crowd at a football game?

13. Estimate the decibel level of each of the sounds in the previous question.

14. A binary star system consists of two stars revolving about their common center of mass. If we observe the light reaching us from one of these stars as it makes one complete revolution, what does the Doppler effect predict will happen to this light?

15. How can an object move with respect to an observer so that the sound from it is not shifted in frequency?

16. Suppose the wind blows. Does this cause a Doppler effect for sound propagating through the air? Is it like a moving source or a moving observer?

17. Why is it not possible to use sonar (sound waves) to determine the speed of an object traveling faster than the speed of sound?

18. Why is it so quiet after a snowfall?

19. Why is the intensity of an echo less than that of the original sound?

20. A loudspeaker built into the exterior wall of an airplane produces a large-amplitude burst of vibration at 200 Hz, then a burst at 300 Hz, and then a burst at 400 Hz (Boop . . . baap . . . beep), all while the plane is flying faster than the speed of sound. Describe qualitatively what an observer hears if she is in front of the plane, close to its flight path. What If? What will the observer hear if the pilot uses the loudspeaker to say, “How are you?”?

21. In several cases, a nearby star has been found to have a large planet orbiting about it, although the planet could not be seen. Using the ideas of a system rotating about its center of mass and of the Doppler shift for light (which is in several ways similar to the Doppler effect for sound), explain how an astronomer could determine the presence of the invisible planet.

### PROBLEMS

1. 2, 3 = straightforward, intermediate, challenging  □ = full solution available in the Student Solutions Manual and Study Guide
   ● = coached solution with hints available at http://www.pse6.com  ☐ = computer useful in solving problem
   = paired numerical and symbolic problems

#### Section 17.1 Speed of Sound Waves

1. Suppose that you hear a clap of thunder 16.2 s after seeing the associated lightning stroke. The speed of sound waves in air is 343 m/s, and the speed of light is $3.00 \times 10^8$ m/s. How far are you from the lightning stroke?

2. Find the speed of sound in mercury, which has a bulk modulus of approximately $2.80 \times 10^{11}$ N/m$^2$ and a density of 15 600 kg/m$^3$.

3. A flowerpot is knocked off a balcony 20.0 m above the sidewalk and falls toward an unsuspecting 1.75-m-tall man who is standing below. How close to the sidewalk can the flower pot fall before it is too late for a warning shouted from the balcony to reach the man in time? Assume that the man below requires 0.300 s to respond to the warning.

4. The speed of sound in air (in m/s) depends on temperature according to the approximate expression

   $$v = 331.5 + 0.607T_C$$

   where $T_C$ is the Celsius temperature. In dry air the temperature decreases about 1°C for every 150 m rise in altitude. (a) Assuming this change is constant up to an altitude of 9 000 m, how long will it take the sound from an airplane flying at 9 000 m to reach the ground on a day when the ground temperature is 30°C? (b) What If? Com-
5. A cowboy stands on horizontal ground between two parallel vertical cliffs. He is not midway between the cliffs. He fires a shot and hears its echoes. The second echo arrives 1.92 s after the first and 1.47 s before the third. Consider only the sound traveling parallel to the ground and reflecting from the cliffs. Take the speed of sound as 340 m/s. (a) What is the distance between the cliffs? (b) What if? If he can hear a fourth echo, how long after the third echo does it arrive?

6. A rescue plane flies horizontally at a constant speed searching for a disabled boat. When the plane is directly above the boat, the boat’s crew blows a loud horn. By the time the plane’s sound detector perceives the horn’s sound, the plane has traveled a distance equal to half its altitude above the ocean. If it takes the sound 2.00 s to reach the plane, determine (a) the speed of the plane and (b) its altitude. Take the speed of sound to be 343 m/s.

Section 17.2 Periodic Sound Waves

**Note:** Use the following values as needed unless otherwise specified: the equilibrium density of air at 20°C is \( \rho = 1.20 \text{ kg/m}^3 \). The speed of sound in air is \( v = 343 \text{ m/s} \). Pressure variations \( \Delta P \) are measured relative to atmospheric pressure, \( 1.013 \times 10^5 \text{ N/m}^2 \). Problem 70 in Chapter 2 can also be assigned with this section.

7. A bat (Fig. P17.7) can detect very small objects, such as an insect whose length is approximately equal to one wavelength of the sound the bat makes. If a bat emits chirps at a frequency of 60.0 kHz, and if the speed of sound in air is 340 m/s, what is the smallest insect the bat can detect?

8. An ultrasonic tape measure uses frequencies above 20 MHz to determine dimensions of structures such as buildings. It does this by emitting a pulse of ultrasound into air and then measuring the time for an echo to return from a reflecting surface whose distance away is to be measured. The distance is displayed as a digital read-out. For a tape measure that emits a pulse of ultrasound with a frequency of 22.0 MHz, (a) What is the distance to an object from which the echo pulse returns after 24.0 ms when the air temperature is 26°C? (b) What should be the duration of the emitted pulse if it is to include 10 cycles of the ultrasonic wave? (c) What is the spatial length of such a pulse?

9. Ultrasound is used in medicine both for diagnostic imaging and for therapy. For diagnosis, short pulses of ultrasound are passed through the patient’s body. An echo reflected from a structure of interest is recorded, and from the time delay for the return of the echo the distance to the structure can be determined. A single transducer emits and detects the ultrasound. An image of the structure is obtained by reducing the data with a computer. With sound of low intensity, this technique is noninvasive and harmless. It is used to examine fetuses, tumors, aneurysms, gallstones, and many other structures. A Doppler ultrasound unit is used to study blood flow and functioning of the heart. To reveal detail, the wavelength of the reflected ultrasound must be small compared to the size of the object reflecting the wave. For this reason, frequencies in the range 1.00 to 20.0 MHz are used. What is the range of wavelengths corresponding to this range of frequencies? The speed of ultrasound in human tissue is about 1500 m/s (nearly the same as the speed of sound in water).

10. A sound wave in air has a pressure amplitude equal to \( 4.00 \times 10^{-3} \text{ N/m}^2 \). Calculate the displacement amplitude of the wave at a frequency of 10.0 kHz.

11. A sinusoidal sound wave is described by the displacement wave function

\[
s(x, t) = (2.00 \text{ mm}) \cos[(15.7 \text{ m}^{-1})x - (858 \text{ s}^{-1})t]
\]

(a) Find the amplitude, wavelength, and speed of this wave. (b) Determine the instantaneous displacement from equilibrium of the elements of air at the position \( x = 0.050 \text{ m} \) at \( t = 3.00 \text{ ms} \). (c) Determine the maximum speed of the element’s oscillatory motion.

12. As a certain sound wave travels through the air, it produces pressure variations (above and below atmospheric pressure) given by \( \Delta P = 1.27 \sin(\pi x - 340 \pi t) \) in SI units. Find (a) the amplitude of the pressure variations, (b) the frequency, (c) the wavelength in air, and (d) the speed of the sound wave.

13. Write an expression that describes the pressure variation as a function of position and time for a sinusoidal sound wave in air, if \( \lambda = 0.100 \text{ m} \) and \( \Delta P_{\text{max}} = 0.200 \text{ N/m}^2 \).

14. Write the function that describes the displacement wave corresponding to the pressure wave in Problem 13.

15. An experimenter wishes to generate in air a sound wave that has a displacement amplitude of \( 5.50 \times 10^{-6} \text{ m} \). The pressure amplitude is to be limited to \( 0.840 \text{ N/m}^2 \). What is the minimum wavelength the sound wave can have?
16. The tensile stress in a thick copper bar is 99.5% of its elastic breaking point of $13.0 \times 10^9$ N/m$^2$. If a 500-Hz sound wave is transmitted through the material, (a) what displacement amplitude will cause the bar to break? (b) What is the maximum speed of the elements of copper at this moment? (c) What is the sound intensity in the bar?

17. Prove that sound waves propagate with a speed given by Equation 17.1. Proceed as follows. In Figure 17.3, consider a thin cylindrical layer of air in the cylinder, with face area $A$ and thickness $\Delta x$. Draw a free-body diagram of this thin layer. Show that $\Sigma F_x = ma_t$ implies that $-\frac{\partial \rho}{\partial x} A \Delta x = \rho A \Delta x \left(\frac{2s^2}{\lambda t^2}\right)$. By substituting $\Delta P = -B(\Delta s / \Delta x)$, obtain the wave equation for sound, $(B/\rho)(\frac{2s^2}{\lambda t^2}) = (\frac{2s^2}{\lambda t^2})$. To a mathematical physicist, this equation demonstrates the existence of sound waves and determines their speed. As a physics student, you must take another step or two. Substitute into the wave equation the trial solution $s(x, t) = s_{max} \cos(\omega t - kx)$. Show that this function satisfies the wave equation provided that $\omega / k = \sqrt{B/\rho}$. This result reveals that sound waves exist provided that they move with the speed $v = \frac{2\pi}{\lambda} / 2\pi = \omega / k = \sqrt{B/\rho}$.

### Section 17.3 Intensity of Periodic Sound Waves

18. The area of a typical cardard is about $5.00 \times 10^{-3}$ m$^2$. Calculate the sound power incident on a cardard at (a) the threshold of hearing and (b) the threshold of pain.  

19. Calculate the sound level in decibels of a sound wave that has an intensity of $4.00 \mu$W/m$^2$.

20. A vacuum cleaner produces sound with a measured sound level of 70.0 dB. (a) What is the intensity of this sound in W/m$^2$? (b) What is the pressure amplitude of the sound?

21. The intensity of a sound wave at a fixed distance from a speaker vibrating at 1.00 kHz is 0.600 W/m$^2$. (a) Determine the intensity if the frequency is increased to 2.50 kHz while a constant displacement amplitude is maintained. (b) Calculate the intensity if the frequency is reduced to 0.500 kHz and the displacement amplitude is doubled.

22. The intensity of a sound wave at a fixed distance from a speaker vibrating at a frequency $f$ is $I$. (a) Determine the intensity if the frequency is increased to $f'$ while a constant displacement amplitude is maintained. (b) Calculate the intensity if the frequency is reduced to $f/2$ and the displacement amplitude is doubled.

23. The most soaring vocal melody is in Johann Sebastian Bach’s *Mass in B minor*. A portion of the score for the Credo section, number 9, bars 25 to 33, appears in Figure P17.23. The repeating syllable “O” in the phrase “resurrectionem mortuorum” (the resurrection of the dead) is seamlessly passed from basses to tenors to altos to first sopranos, like a baton in a relay. Each voice carries the melody up in a run of an octave or more. Together they carry it from D below middle C to A above a tenor’s high C. In concert pitch, these notes are now assigned frequencies of 148.6 Hz and 880.0 Hz. (a) Find the wavelengths of the initial and final notes. (b) Assume that the choir sings the melody with a uniform sound level of 75.0 dB. Find the pressure amplitudes of the initial and final notes. (c) Find the displacement amplitudes of the initial and final notes. (d) What If? In Bach’s time, before the invention of the tuning fork, frequencies were assigned to notes as a matter of immediate local convenience. Assume that the rising melody was sung starting from 134.8 Hz and ending at 804.9 Hz. How would the answers to parts (a) through (c) change?

24. The tube depicted in Figure 17.2 is filled with air at 20°C and equilibrium pressure 1 atm. The diameter of the tube is 8.00 cm. The piston is driven at a frequency of 600 Hz with an amplitude of 0.120 mm. What power must be supplied to maintain the oscillation of the piston?

25. A family ice show is held at an enclosed arena. The skaters perform to music with level 80.0 dB. This is too loud for your baby, who yells at 75.0 dB. (a) What total sound intensity engulfs you? (b) What is the combined sound level?

26. Consider sinusoidal sound waves propagating in these three different media: air at 0°C, water, and iron. Use densities and speeds from Tables 14.1 and 17.1. Each wave has the same intensity $I_0$ and the same angular frequency $\omega_0$. (a) Compare the values of the wavelength in the three media. (b) Compare the values of the displacement amplitude in the three media. (c) Compare the values of the pressure amplitude in the three media. (d) For values of $\omega_0 = 2 000 \pi$ rad/s and $I_0 = 1.00 \times 10^{-6}$ W/m$^2$, evaluate the wavelength, displacement amplitude, and pressure amplitude in each of the three media.

27. The power output of a certain public address speaker is 6.00 W. Suppose it broadcasts equally in all directions. (a) Within what distance from the speaker would the sound be painful to the ear? (b) At what distance from the speaker would the sound be barely audible?

**Figure P17.23** Bass (blue), tenor (green), alto (brown), and first soprano (red) parts for a portion of Bach’s *Mass in B minor*. For emphasis, the line we choose to call the melody is printed in black. Parts for the second soprano, violins, viola, flutes, oboes, and continuo are omitted. The tenor part is written as it is sung.
28. Show that the difference between decibel levels $\beta_1$ and $\beta_2$ of a sound is related to the ratio of the distances $r_1$ and $r_2$ from the sound source by

$$\beta_2 - \beta_1 = 20 \log \left( \frac{r_1}{r_2} \right)$$

29. A firework charge is detonated many meters above the ground. At a distance of 400 m from the explosion, the acoustic pressure reaches a maximum of 10.0 N/m². Assume that the speed of sound is constant at 343 m/s throughout the atmosphere over the region considered, that the ground absorbs all the sound falling on it, and that the air absorbs sound energy as described by the rate 7.00 dB/km. What is the sound level (in dB) at 4.00 km from the explosion?

30. A loudspeaker is placed between two observers who are 110 m apart, along the line connecting them. If one observer records a sound level of 60.0 dB and the other records a sound level of 80.0 dB, how far is the speaker from each observer?

31. Two small speakers emit sound waves of different frequencies. Speaker A has an output of 1.00 mW, and speaker B has an output of 1.50 mW. Determine the sound level (in dB) at point C (Fig. P17.31) if (a) only speaker A emits sound, (b) only speaker B emits sound, and (c) both speakers emit sound.

![Figure P17.31](image)

32. A jackhammer, operated continuously at a construction site, behaves as a point source of spherical sound waves. A construction supervisor stands 50.0 m due north of this sound source and begins to walk due west. How far does she have to walk in order for the amplitude of the wave function to drop by a factor of 2.00?

33. The sound level at a distance of 3.00 m from a source is 120 dB. At what distance will the sound level be (a) 100 dB and (b) 10.0 dB?

34. A fireworks rocket explodes at a height of 100 m above the ground. An observer on the ground directly under the explosion experiences an average sound intensity of $7.00 \times 10^{-2}$ W/m² for 0.200 s. (a) What is the total sound energy of the explosion? (b) What is the sound level in decibels heard by the observer?

35. As the people sing in church, the sound level everywhere inside is 101 dB. No sound is transmitted through the massive walls, but all the windows and doors are open on a summer morning. Their total area is 22.0 m². (a) How much sound energy is radiated in 20.0 min? (b) Suppose the ground is a good reflector and sound radiates uniformly in all horizontal and upward directions. Find the sound level 1 km away.

36. The smallest change in sound level that a person can distinguish is approximately 1 dB. When you are standing next to your power lawnmower as it is running, can you hear the steady roar of your neighbor’s lawnmower? Perform an order-of-magnitude calculation to substantiate your answer, stating the data you measure or estimate.

### Section 17.4 The Doppler Effect

37. A train is moving parallel to a highway with a constant speed of 20.0 m/s. A car is traveling in the same direction as the train with a speed of 40.0 m/s. The car horn sounds at a frequency of 510 Hz, and the train whistle sounds at a frequency of 320 Hz. (a) When the car is behind the train, what frequency does an occupant of the car observe for the train whistle? (b) After the car passes and is in front of the train, what frequency does a train passenger observe for the car horn?

38. Expectant parents are thrilled to hear their unborn baby’s heartbeat, revealed by an ultrasonic motion detector. Suppose the fetus’s ventricular wall moves in simple harmonic motion with an amplitude of 1.80 mm and a frequency of 115 per minute. (a) Find the maximum linear speed of the heart wall. Suppose the motion detector in contact with the mother’s abdomen produces sound at 2 000 000.0 Hz, which travels through tissue at 1.50 km/s. (b) Find the maximum frequency at which sound arrives at the wall of the baby’s heart. (c) Find the maximum frequency at which reflected sound is received by the motion detector. By electronically “listening” for echoes at a frequency different from the broadcast frequency, the motion detector can produce beeps of audible sound in synchronization with the fetal heartbeat.

39. Standing at a crosswalk, you hear a frequency of 560 Hz from the siren of an approaching ambulance. After the ambulance passes, the observed frequency of the siren is 480 Hz. Determine the ambulance’s speed from these observations.

40. A block with a speaker bolted to it is connected to a spring having spring constant $k = 20.0$ N/m as in Figure P17.40. The total mass of the block and speaker is 5.00 kg, and the amplitude of this unit’s motion is 0.500 m. (a) If the speaker emits sound waves of frequency 440 Hz, determine the highest and lowest frequencies heard by the person to the right of the speaker. (b) If the maximum sound level heard by the person is 60.0 dB when he is closest to the
speaker, 1.00 m away, what is the minimum sound level heard by the observer? Assume that the speed of sound is 343 m/s.

A tuning fork vibrating at 512 Hz falls from rest and accelerates at 9.80 m/s². How far below the point of release is the tuning fork when waves of frequency 485 Hz reach the release point? Take the speed of sound in air to be 340 m/s.

At the Winter Olympics, an athlete rides her luge down the track while a bell just above the wall of the chute rings continuously. When her sled passes the bell, she hears the frequency of the bell fall by the musical interval called a minor third. That is, the frequency she hears drops to five sixths of its original value. (a) Find the speed of sound in air at the ambient temperature −10.0°C. (b) Find the speed of the athlete.

A siren mounted on the roof of a firehouse emits sound at a frequency of 900 Hz. A steady wind is blowing with a speed of 12.5 m/s. Taking the speed of sound in calm air to be 343 m/s, find the wavelength of the sound (a) upwind of the siren and (b) downwind of the siren. Firefighters are approaching the siren from various directions at 15.0 m/s. What frequency does a firefighter hear (c) if he or she is approaching from an upwind position, so that he or she is moving in the direction in which the wind is blowing? (d) if he or she is approaching from a downwind position and moving against the wind?

The Concorde can fly at Mach 1.50, which means the speed of the plane is 1.50 times the speed of sound in air. What is the angle between the direction of propagation of the shock wave and the direction of the plane’s velocity?

When high-energy charged particles move through a transparent medium with a speed greater than the speed of light in that medium, a shock wave, or bow wave, of light is produced. This phenomenon is called the Cerenkov effect. When a nuclear reactor is shielded by a large pool of water, Cerenkov radiation can be seen as a blue glow in the vicinity of the reactor core, due to high-speed electrons moving through the water. In a particular case, the Cerenkov radiation produces a wave front with an apex half-angle of 53.0°. Calculate the speed of the electrons in the water. (The speed of light in water is $2.25 \times 10^8$ m/s.)

The loop of a circus ringmaster’s whip travels at Mach 1.38 (that is, $v_S/v = 1.38$). What angle does the shock wave make with the direction of the whip’s motion?

A supersonic jet traveling at Mach 3.00 at an altitude of 20 000 m is directly over a person at time $t = 0$ as in Figure P17.47. (a) How long will it be before the person encounters the shock wave? (b) Where will the plane be when it is finally heard? (Assume the speed of sound in air is 335 m/s.)

Section 17.5 Digital Sound Recording

Section 17.6 Motion Picture Sound

This problem represents a possible (but not recommended) way to code instantaneous pressures in a sound wave into 16-bit digital words. Example 17.2 mentions that the pressure amplitude of a 120 dB sound is 28.7 N/m². Let this pressure variation be represented by the digital code 65 536. Let zero pressure variation be represented on the recording by the digital word 0. Let other intermediate pressures be represented by digital words of intermediate size, in direct proportion to the pressure. (a) What digital word would represent the maximum pressure in a 40 dB sound? (b) Explain why this scheme works poorly for soft sounds. (c) Explain how this coding scheme would clip off half of the waveform of any sound, ignoring the actual shape of the wave and turning it into a string of zeros. By introducing sharp corners into every recorded waveform, this coding scheme would make everything sound like a buzzer or a kazoo.

Only two recording channels are required to give the illusion of sound coming from any point located between two speakers of a stereophonic sound system. If the same signal is recorded in both channels, a listener will hear it coming from a single direction halfway between the two speakers. This “phantom orchestra” illusion can be heard in the two-channel original Broadway cast recording of the song “Do-Re-Mi” from The Sound of Music (Columbia Records KOS 2020). Each of the eight singers can be heard at a different location between the loudspeakers. All listeners with normal hearing will agree on their locations. The brain can sense the direction of sound by noting how
much earlier a sound is heard in one ear than in the other. Model your ears as two sensors 19.0 cm apart in a flat screen. If a click from a distant source is heard 210 μs earlier in the left ear than in the right, from what direction does it appear to originate?

50. Assume that a loudspeaker broadcasts sound equally in all directions and produces sound with a level of 103 dB at a distance of 1.60 m from its center. (a) Find its sound power output. (b) If the salesperson claims to be giving you 150 W per channel, he is referring to the electrical power input to the speaker. Find the efficiency of the speaker—that is, the fraction of input power that is converted into useful output power.

Additional Problems

51. A large set of unoccupied football bleachers has solid seats and risers. You stand on the field in front of the bleachers and fire a starter’s pistol or sharply clap two wooden boards together once. The sound pulse you produce has no definite frequency and no wavelength. The sound you hear reflected from the bleachers has an identifiable frequency and may remind you of a short toot on a trumpet, or of a buzzer or kazoo. Account for this sound. Compute order-of-magnitude estimates for its frequency, wavelength, and duration, on the basis of data you specify.

52. Many artists sing very high notes in ad lib ornaments and cadenzas. The highest note written for a singer in a published score was F-sharp above high C, 1.480 kHz, for Zerbinetta in the original version of Richard Strauss’s opera Ariadne auf Naxos. (a) Find the wavelength of this sound in air. (b) Suppose people in the fourth row of seats hear this note with level 81.0 dB. Find the displacement amplitude of the sound. (c) What If? Because of complaints, Strauss later transposed the note down to F above high C, 1.397 kHz. By what increment did the wavelength change?

53. A sound wave in a cylinder is described by Equations 17.2 through 17.4. Show that

\[ \Delta P = \frac{2P_0 \nu}{\sqrt{\nu^2 - \nu_0^2}} \]

54. On a Saturday morning, pickup trucks and sport utility vehicles carrying garbage to the town dump form a nearly steady procession on a country road, all traveling at 19.7 m/s. From one direction, two trucks arrive at the dump every 5 min. A bicyclist is also traveling toward the dump, at 4.47 m/s. (a) With what frequency do the trucks pass him? (b) What If? A hill does not slow down the trucks, but makes the out-of-shape cyclist’s speed drop to 1.56 m/s. How often do noisy, smelly, inefficient, garbage-dripping, roadhoggling trucks whiz past him now?

55. The ocean floor is underlain by a layer of basalt that constitutes the crust, or uppermost layer, of the Earth in that region. Below this crust is found denser periodotite rock, which forms the Earth’s mantle. The boundary between these two layers is called the Mohorovicic discontinuity (“Moho” for short). If an explosive charge is set off at the surface of the basalt, it generates a seismic wave that is reflected back out at the Moho. If the speed of this wave in basalt is 6.50 km/s and the two-way travel time is 1.85 s, what is the thickness of this oceanic crust?

56. For a certain type of steel, stress is always proportional to strain with Young’s modulus as shown in Table 12.1. The steel has the density listed for iron in Table 14.1. It will fail by bending permanently if subjected to compressive stress greater than its yield strength \( \sigma_y = 400 \text{ MPa} \). A rod 80.0 cm long, made of this steel, is fired at 12.0 m/s straight at a very hard wall, or at another identical rod moving in the opposite direction. (a) The speed of a one-dimensional compressional wave moving along the rod is given by \( \sqrt{\nu^2 - \nu_0^2} \), where \( \rho \) is the density and \( Y \) is Young’s modulus for the rod. Calculate this speed. (b) After the front end of the rod hits the wall and stops, the back end of the rod keeps moving, as described by Newton’s first law, until it is stopped by excess pressure in a sound wave moving back through the rod. How much time elapses before the back end of the rod receives the message that it should stop? (c) How far has the back end of the rod moved in this time? Find (d) the strain in the rod and (e) the stress. (f) If it is not to fail, show that the maximum impact speed a rod can have is given by the expression \( \nu / Y / \rho \).

57. To permit measurement of her speed, a skydiver carries a buzzer emitting a steady tone at 1 800 Hz. A friend on the ground at the landing site directly below listens to the amplified sound he receives. Assume that the air is calm and that the sound speed is 343 m/s, independent of altitude. While the skydiver is falling at terminal speed, her friend on the ground receives waves of frequency 2 150 Hz. (a) What is the skydiver’s speed of descent? (b) What If? Suppose the skydiver can hear the sound of the buzzer reflected from the ground. What frequency does she receive?

58. A train whistle (\( f = 400 \text{ Hz} \)) sounds higher or lower in frequency depending on whether it approaches or recedes. (a) Prove that the difference in frequency between the approaching and receding train whistle is

\[ \Delta f = \frac{2u/v}{1 - u^2/v^2} f \]

where \( u \) is the speed of the train and \( v \) is the speed of sound. (b) Calculate this difference for a train moving at a speed of 130 km/h. Take the speed of sound in air to be 340 m/s.

59. Two ships are moving along a line due east. The trailing vessel has a speed relative to a land-based observation point of 64.0 km/h, and the leading ship has a speed of 45.0 km/h relative to that point. The two ships are in a region of the ocean where the current is moving uniformly due west at 10.0 km/h. The trailing ship transmits a sonar signal at a frequency of 1 200.0 Hz. What frequency is monitored by the leading ship? (Use 1 520 m/s as the speed of sound in ocean water.)

60. A bat, moving at 5.00 m/s, is chasing a flying insect (Fig. P17.7). If the bat emits a 40.0 kHz chirp and receives back an echo at 40.4 kHz, at what speed is the insect moving toward or away from the bat? (Take the speed of sound in air to be \( v = 340 \text{ m/s} \).)

61. A supersonic aircraft is flying parallel to the ground. When the aircraft is directly overhead, an observer sees a rocket fired from the aircraft. Ten seconds later the observer
hears the sonic boom, followed 2.80 s later by the sound of the rocket engine. What is the Mach number of the aircraft?

62. A police car is traveling east at 40.0 m/s along a straight road, overtaking a car ahead of it moving east at 30.0 m/s. The police car has a malfunctioning siren that is stuck at 1,000 Hz. (a) Sketch the appearance of the wave fronts of the sound produced by the siren. Show the wave fronts both to the east and to the west of the police car. (b) What would be the wavelength in air of the siren sound if the police car were at rest? (c) What is the wavelength in front of the police car? (d) What is behind the police car? (e) What is the frequency heard by the driver being chased?

63. The speed of a one-dimensional compressional wave traveling along a thin copper rod is 3.56 km/s. A copper bar is given a sharp compressional blow at one end. The sound of the blow, traveling through air at 0°C, reaches the opposite end of the bar 6.40 ms later than the sound transmitted through the metal of the bar. What is the length of the bar?

64. A jet flies toward higher altitude at a constant speed of 1,963 m/s in a direction making an angle θ with the horizontal (Fig. P17.64). An observer on the ground hears the jet for the first time when it is directly overhead. Determine the value of θ if the speed of sound in air is 340 m/s.

65. A meteoroid the size of a truck enters the earth’s atmosphere at a speed of 20.0 km/s and is not significantly slowed before entering the ocean. (a) What is the Mach angle of the shock wave from the meteoroid in the atmosphere? (Use 331 m/s as the sound speed.) (b) Assuming that the meteoroid survives the impact with the ocean surface, what is the (initial) Mach angle of the shock wave that the meteoroid produces in the water? (Use the wave speed for seawater given in Table 17.1.)

66. An interstate highway has been built through a poor neighborhood in a city. In the afternoon, the sound level in a rented room is 80.0 dB, as 100 cars pass outside the window every minute. Late at night, when the tenant is working in a factory, the traffic flow is only five cars per minute. What is the average late-night sound level?

67. With particular experimental methods, it is possible to produce and observe in a long thin rod both a longitudinal wave and a transverse wave whose speed depends primarily on tension in the rod. The speed of the longitudinal wave is determined by the Young’s modulus and the density of the material as \( \sqrt{Y/\rho} \). The transverse wave can be modeled as a wave in a stretched string. A particular metal rod is 150 cm long and has a radius of 0.200 cm and a mass of 50.9 g. Young’s modulus for the material is 6.80 \( \times 10^10 \) N/m². What must the tension in the rod be if the ratio of the speed of longitudinal waves to the speed of transverse waves is 8.00?

68. A siren creates sound with a level β at a distance d from the speaker. The siren is powered by a battery that delivers a total energy E. Let \( \epsilon \) represent the efficiency of the siren. (That is, \( \epsilon \) is equal to the output sound energy divided by the supplied energy). Determine the total time the siren can sound.

69. The Doppler equation presented in the text is valid when the motion between the observer and the source occurs on a straight line, so that the source and observer are moving either directly toward or directly away from each other. If this restriction is relaxed, one must use the more general Doppler equation

\[
\frac{\nu}{\nu_0} = \frac{\nu_0 \cos \theta - \nu \cos \theta'}{\nu_0 \cos \theta - \nu \cos \theta'}
\]

where \( \theta \) and \( \theta' \) are defined in Figure P17.69a. (a) Show that if the observer and source are moving away from each other, the preceding equation reduces to Equation 17.13 with negative values for both \( \nu_0 \) and \( \nu \). (b) Use the preceding equation to solve the following problem. A train moves at a constant speed of 25.0 m/s toward the intersection shown in Figure P17.69b. A car is stopped near the intersection, 30.0 m from the tracks. If the train’s horn emits a frequency of 500 Hz, what is the frequency heard by the passengers in the car when the train is 40.0 m from the intersection? Take the speed of sound to be 343 m/s.

70. Equation 17.7 states that, at distance \( r \) away from a point source with power \( \mathcal{P}_{av} \), the wave intensity is

\[
I = \frac{\mathcal{P}_{av}}{4\pi r^2}
\]

Study Figure 17.9 and prove that, at distance \( r \) straight in front of a point source with power \( \mathcal{P}_{av} \) moving with...
constant speed $v_S$, the wave intensity is

$$I = \frac{\rho v}{4\pi r^2} \left( \frac{v - v_S}{v} \right)$$

Three metal rods are located relative to each other as shown in Figure P17.71, where $L_1 + L_2 = L_3$. The speed of sound in a rod is given by $v = \sqrt{Y/\rho}$, where $\rho$ is the density and $Y$ is Young’s modulus for the rod. Values of density and Young’s modulus for the three materials are

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Young’s Modulus $Y$ (N/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.70 x $10^3$</td>
<td>7.00 x $10^10$</td>
</tr>
<tr>
<td>2</td>
<td>11.3 x $10^3$</td>
<td>1.60 x $10^10$</td>
</tr>
<tr>
<td>3</td>
<td>11.0 x $10^3$</td>
<td>1.60 x $10^10$</td>
</tr>
</tbody>
</table>

(a) If $L_3 = 1.50$ m, what must the ratio $L_1/L_2$ be if a sound wave is to travel the length of rods 1 and 2 in the same time as it takes for the wave to travel the length of rod 3? (b) If the frequency of the source is 4.00 kHz, determine the phase difference between the wave traveling along rods 1 and 2 and the one traveling along rod 3.

![Figure P17.71](image)

The smallest wavelength possible for a sound wave in air is on the order of the separation distance between air molecules. Find the order of magnitude of the highest-frequency sound wave possible in air, assuming a wave speed of 343 m/s, density 1.20 kg/m$^3$, and an average molecular mass of 4.82 x $10^{-26}$ kg.

Answers to Quick Quizzes

17.1 (c). Although the speed of a wave is given by the product of its wavelength (a) and frequency (b), it is not affected by changes in either one. The amplitude (d) of a sound wave determines the size of the oscillations of elements of air but does not affect the speed of the wave through the air.

17.2 (c). Because the bottom of the bottle is a rigid barrier, the displacement of elements of air at the bottom is zero. Because the pressure variation is a minimum or a maximum when the displacement is zero, and the pulse is moving downward, the pressure variation at the bottom is a maximum.

17.3 (c). The ear trumpet collects sound waves from the large area of its opening and directs it toward the ear. Most of the sound in this large area would miss the ear in the absence of the trumpet.

17.4 (b). The large area of the guitar body sets many elements of air into oscillation and allows the energy to leave the system by mechanical waves at a much larger rate than from the thin vibrating string.

17.5 (c). The only parameter that adds directly is intensity. Because of the logarithm function in the definition of sound level, sound levels cannot be added directly.

17.6 (b). The factor of 100 is two powers of ten. Thus, the logarithm of 100 is 2, which multiplied by 10 gives 20 dB.

17.7 (e). The wave speed cannot be changed by moving the source, so (a) and (b) are incorrect. The detected wavelength is largest at A, so (c) and (d) are incorrect. Choice (f) is incorrect because the detected frequency is lowest at location A.

17.8 (e). The intensity of the sound increases because the train is moving closer to you. Because the train moves at a constant velocity, the Doppler-shifted frequency remains fixed.

17.9 (b). The Mach number is the ratio of the plane’s speed (which does not change) to the speed of sound, which is greater in the warm air than in the cold. The denominator of this ratio increases while the numerator stays constant. Therefore, the ratio as a whole—the Mach number—decreases.
Superposition and Standing Waves

Guitarist Carlos Santana takes advantage of standing waves on strings. He changes to a higher note on the guitar by pushing the strings against the frets on the fingerboard, shortening the lengths of the portions of the strings that vibrate. (Bettmann/Corbis)
In the previous two chapters, we introduced the wave model. We have seen that waves are very different from particles. A particle is of zero size, while a wave has a characteristic size—the wavelength. Another important difference between waves and particles is that we can explore the possibility of two or more waves combining at one point in the same medium. We can combine particles to form extended objects, but the particles must be at different locations. In contrast, two waves can both be present at the same location, and the ramifications of this possibility are explored in this chapter.

When waves are combined, only certain allowed frequencies can exist on systems with boundary conditions—the frequencies are quantized. Quantization is a notion that is at the heart of quantum mechanics, a subject that we introduce formally in Chapter 40. There we show that waves under boundary conditions explain many of the quantum phenomena. For our present purposes in this chapter, quantization enables us to understand the behavior of the wide array of musical instruments that are based on strings and air columns.

We also consider the combination of waves having different frequencies and wavelengths. When two sound waves having nearly the same frequency interfere, we hear variations in the loudness called beats. The beat frequency corresponds to the rate of alternation between constructive and destructive interference. Finally, we discuss how any nonsinusoidal periodic wave can be described as a sum of sine and cosine functions.

18.1 Superposition and Interference

Many interesting wave phenomena in nature cannot be described by a single traveling wave. Instead, one must analyze complex waves in terms of a combination of traveling waves. To analyze such wave combinations, one can make use of the superposition principle:

If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.

Waves that obey this principle are called linear waves. In the case of mechanical waves, linear waves are generally characterized by having amplitudes much smaller than their wavelengths. Waves that violate the superposition principle are called nonlinear waves and are often characterized by large amplitudes. In this book, we deal only with linear waves.

One consequence of the superposition principle is that two traveling waves can pass through each other without being destroyed or even altered. For instance,
when two pebbles are thrown into a pond and hit the surface at different places, the expanding circular surface waves do not destroy each other but rather pass through each other. The complex pattern that is observed can be viewed as two independent sets of expanding circles. Likewise, when sound waves from two sources move through air, they pass through each other.

Figure 18.1 is a pictorial representation of the superposition of two pulses. The wave function for the pulse moving to the right is $y_1$, and the wave function for the pulse moving to the left is $y_2$. The pulses have the same speed but different shapes, and the displacement of the elements of the medium is in the positive $y$ direction for both pulses. When the waves begin to overlap (Fig. 18.1b), the wave function for the resulting complex wave is given by $y_1 + y_2$. When the crests of the pulses coincide (Fig. 18.1c), the resulting wave given by $y_1 + y_2$ has a larger amplitude than that of the individual pulses. The two pulses finally separate and continue moving in their original directions (Fig. 18.1d). Note that the pulse shapes remain unchanged after the interaction, as if the two pulses had never met!

The combination of separate waves in the same region of space to produce a resultant wave is called interference. For the two pulses shown in Figure 18.1, the displacement of the elements of the medium is in the positive $y$ direction for both pulses, and the resultant pulse (created when the individual pulses overlap) exhibits an amplitude greater than that of either individual pulse. Because the displacements caused by the two pulses are in the same direction, we refer to their superposition as constructive interference.

Now consider two pulses traveling in opposite directions on a taut string where one pulse is inverted relative to the other, as illustrated in Figure 18.2. In this case, when the pulses begin to overlap, the resultant pulse is given by $y_1 + y_2$, but the values of the function $y_2$ are negative. Again, the two pulses pass through each other; however, because the displacements caused by the two pulses are in opposite directions, we refer to their superposition as destructive interference.
Active Figure 18.2 (a–e) Two pulses traveling in opposite directions and having displacements that are inverted relative to each other. When the two overlap in (c), their displacements partially cancel each other. (f) Photograph of the superposition of two symmetric pulses traveling in opposite directions, where one is inverted relative to the other.

Quick Quiz 18.1 Two pulses are traveling toward each other, each at 10 cm/s on a long string, as shown in Figure 18.3. Sketch the shape of the string at \( t = 0.6 \) s.

Quick Quiz 18.2 Two pulses move in opposite directions on a string and are identical in shape except that one has positive displacements of the elements of the string and the other has negative displacements. At the moment that the two pulses completely overlap on the string, (a) the energy associated with the pulses has disappeared (b) the string is not moving (c) the string forms a straight line (d) the pulses have vanished and will not reappear.
Superposition of Sinusoidal Waves

Let us now apply the principle of superposition to two sinusoidal waves traveling in the same direction in a linear medium. If the two waves are traveling to the right and have the same frequency, wavelength, and amplitude but differ in phase, we can express their individual wave functions as

\[
y_1 = A \sin(kx - \omega t) \quad \quad y_2 = A \sin(kx - \omega t + \phi)
\]

where, as usual, \( k = 2\pi/\lambda \), \( \omega = 2\pi f \), and \( \phi \) is the phase constant, which we discussed in Section 16.2. Hence, the resultant wave function \( y \) is

\[
y = y_1 + y_2 = A [\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]
\]

To simplify this expression, we use the trigonometric identity

\[
\sin a + \sin b = 2 \cos \left( \frac{a - b}{2} \right) \sin \left( \frac{a + b}{2} \right)
\]

If we let \( a = kx - \omega t \) and \( b = kx - \omega t + \phi \), we find that the resultant wave function \( y \) reduces to

\[
y = 2A \cos \left( \frac{\phi}{2} \right) \sin \left( kx - \omega t + \frac{\phi}{2} \right)
\]

This result has several important features. The resultant wave function \( y \) also is sinusoidal and has the same frequency and wavelength as the individual waves because the sine function incorporates the same values of \( k \) and \( \omega \) that appear in the original wave functions. The amplitude of the resultant wave is \( 2A \cos(\phi/2) \), and its phase is \( \phi/2 \). If the phase constant \( \phi \) equals 0, then \( \cos(\phi/2) = \cos 0 = 1 \), and the amplitude of the resultant wave is \( 2A \)—twice the amplitude of either individual wave. In this case the waves are said to be everywhere in phase and thus interfere constructively. That is, the crests and troughs of the individual waves \( y_1 \) and \( y_2 \) occur at the same positions and combine to form the red curve \( y \) of amplitude \( 2A \) shown in Figure 18.4a. Because the

---

**Active Figure 18.4** The superposition of two identical waves \( y_1 \) and \( y_2 \) (blue and green) to yield a resultant wave (red). (a) When \( y_1 \) and \( y_2 \) are in phase, the result is constructive interference. (b) When \( y_1 \) and \( y_2 \) are \( \pi \) rad out of phase, the result is destructive interference. (c) When the phase angle has a value other than 0 or \( \pi \) rad, the resultant wave \( y \) falls somewhere between the extremes shown in (a) and (b).
individual waves are in phase, they are indistinguishable in Figure 18.4a, in which they appear as a single blue curve. In general, constructive interference occurs when \( \cos(\phi/2) = \pm 1 \). This is true, for example, when \( \phi = 0, 2\pi, 4\pi, \ldots \) rad—that is, when \( \phi \) is an even multiple of \( \pi \).

When \( \phi \) is equal to \( \pi \) rad or to any odd multiple of \( \pi \), then \( \cos(\phi/2) = \cos(\pi/2) = 0 \), and the crests of one wave occur at the same positions as the troughs of the second wave (Fig. 18.4b). Thus, the resultant wave has zero amplitude everywhere, as a consequence of destructive interference. Finally, when the phase constant has an arbitrary value other than 0 or an integer multiple of \( \pi \) rad (Fig. 18.4c), the resultant wave has an amplitude whose value is somewhere between 0 and 2A.

### Interference of Sound Waves

One simple device for demonstrating interference of sound waves is illustrated in Figure 18.5. Sound from a loudspeaker S is sent into a tube at point P, where there is a T-shaped junction. Half of the sound energy travels in one direction, and half travels in the opposite direction. Thus, the sound waves that reach the receiver R can travel along either of the two paths. The distance along any path from speaker to receiver is called the path length \( r \). The lower path length \( r_1 \) is fixed, but the upper path length \( r_2 \) can be varied by sliding the U-shaped tube, which is similar to that on a slide trombone. When the difference in the path lengths \( \Delta r = |r_2 - r_1| \) is either zero or some integer multiple of the wavelength \( \lambda \) (that is, \( \Delta r = n\lambda \), where \( n = 0, 1, 2, 3, \ldots \)), the two waves reaching the receiver at any instant are in phase and interfere constructively, as shown in Figure 18.4a. For this case, a maximum in the sound intensity is detected at the receiver. If the path length \( r_2 \) is adjusted such that the path difference \( \Delta r = \lambda/2, 3\lambda/2, \ldots, n\lambda/2 \) (for \( n \) odd), the two waves are exactly \( \pi \) rad, or 180°, out of phase at the receiver and hence cancel each other. In this case of destructive interference, no sound is detected at the receiver. This simple experiment demonstrates that a phase difference may arise between two waves generated by the same source when they travel along paths of unequal lengths. This important phenomenon will be indispensable in our investigation of the interference of light waves in Chapter 37.

It is often useful to express the path difference in terms of the phase angle \( \phi \) between the two waves. Because a path difference of one wavelength corresponds to a phase angle of \( 2\pi \) rad, we obtain the ratio \( \phi/2\pi = \Delta r/\lambda \) or

\[
\Delta r = \frac{\phi}{2\pi} \lambda \tag{18.1}
\]

Using the notion of path difference, we can express our conditions for constructive and destructive interference in a different way. If the path difference is any even multiple of \( \lambda/2 \), then the phase angle \( \phi = 2n\pi \), where \( n = 0, 1, 2, 3, \ldots \), and the interference is constructive. For path differences of odd multiples of \( \lambda/2 \), \( \phi = (2n + 1)\pi \), where \( n = 0, 1, 2, 3, \ldots \), and the interference is destructive. Thus, we have the conditions

---

**Figure 18.5** An acoustical system for demonstrating interference of sound waves. A sound wave from the speaker (S) propagates into the tube and splits into two parts at point P. The two waves, which combine at the opposite side, are detected at the receiver (R). The upper path length \( r_2 \) can be varied by sliding the upper section.
\[ \Delta r = (2n) \frac{\lambda}{2} \quad \text{for constructive interference} \]

and

\[ \Delta r = (2n + 1) \frac{\lambda}{2} \quad \text{for destructive interference} \]

This discussion enables us to understand why the speaker wires in a stereo system should be connected properly. When connected the wrong way—that is, when the positive (or red) wire is connected to the negative (or black) terminal on one of the speakers and the other is correctly wired—the speakers are said to be “out of phase”—one speaker cone moves outward while the other moves inward. As a consequence, the sound wave coming from one speaker destructively interferes with the wave coming from the other—along a line midway between the two, a rarefaction region due to one speaker is superposed on a compression region from the other speaker. Although the two sounds probably do not completely cancel each other (because the left and right stereo signals are usually not identical), a substantial loss of sound quality occurs at points along this line.

**Example 18.1 Two Speakers Driven by the Same Source**

A pair of speakers placed 3.00 m apart are driven by the same oscillator (Fig. 18.6). A listener is originally at point O, which is located 8.00 m from the center of the line connecting the two speakers. The listener then walks to point P, which is a perpendicular distance 0.350 m from O, before reaching the first minimum in sound intensity. What is the frequency of the oscillator?

**Solution** To find the frequency, we must know the wavelength of the sound coming from the speakers. With this information, combined with our knowledge of the speed of sound, we can calculate the frequency. The wavelength can be determined from the interference information given. The first minimum occurs when the two waves reaching the listener at point P are 180° out of phase—in other words, when their path difference \( \Delta r \) equals \( \lambda/2 \). To calculate the path difference, we must first find the path lengths \( r_1 \) and \( r_2 \).

Figure 18.6 shows the physical arrangement of the speakers, along with two shaded right triangles that can be drawn on the basis of the lengths described in the problem. From these triangles, we find that the path lengths are

\[ r_1 = \sqrt{(8.00 \text{ m})^2 + (1.15 \text{ m})^2} = 8.08 \text{ m} \]

and

\[ r_2 = \sqrt{(8.00 \text{ m})^2 + (1.85 \text{ m})^2} = 8.21 \text{ m} \]

Hence, the path difference is \( r_2 - r_1 = 0.13 \text{ m} \). Because we require that this path difference be equal to \( \lambda/2 \) for the first minimum, we find that \( \lambda = 0.26 \text{ m} \).

To obtain the oscillator frequency, we use Equation 16.12, \( v = \lambda f \), where \( v \) is the speed of sound in air, 343 m/s:

\[ f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{0.26 \text{ m}} = 1.3 \text{ kHz} \]

**What If?** What if the speakers were connected out of phase? What happens at point \( P \) in Figure 18.6?

**Answer** In this situation, the path difference of \( \lambda/2 \) combines with a phase difference of \( \lambda/2 \) due to the incorrect wiring to give a full phase difference of \( \lambda \). As a result, the waves are in phase and there is a maximum intensity at point \( P \).

### 18.2 Standing Waves

The sound waves from the speakers in Example 18.1 leave the speakers in the forward direction, and we considered interference at a point in front of the speakers. Suppose that we turn the speakers so that they face each other and then have them emit sound of the same frequency and amplitude. In this situation, two identical waves travel in
opposite directions in the same medium, as in Figure 18.7. These waves combine in accordance with the superposition principle.

We can analyze such a situation by considering wave functions for two transverse sinusoidal waves having the same amplitude, frequency, and wavelength but traveling in opposite directions in the same medium:

\[ y_1 = A \sin(kx - \omega t) \quad y_2 = A \sin(kx + \omega t) \]

where \( y_1 \) represents a wave traveling in the \(+x\) direction and \( y_2 \) represents one traveling in the \(-x\) direction. Adding these two functions gives the resultant wave function \( y \):

\[ y = y_1 + y_2 = A \sin(kx - \omega t) + A \sin(kx + \omega t) \]

When we use the trigonometric identity \( \sin(a \pm b) = \sin(a) \cos(b) \pm \cos(a) \sin(b) \), this expression reduces to

\[ y = (2A \sin kx) \cos \omega t \]

Equation 18.3 represents the wave function of a **standing wave**. A standing wave, such as the one shown in Figure 18.8, is an oscillation pattern **with a stationary outline** that results from the superposition of two identical waves traveling in opposite directions.

Notice that Equation 18.3 does not contain a function of \( kx - \omega t \). Thus, it is not an expression for a traveling wave. If we observe a standing wave, we have no sense of motion in the direction of propagation of either of the original waves. If we compare this equation with Equation 15.6, we see that Equation 18.3 describes a special kind of simple harmonic motion. Every element of the medium oscillates in simple harmonic motion with the same frequency \( \omega \) (according to the \( \cos \omega t \) factor in the equation). However, the amplitude of the simple harmonic motion of a given element (given by the factor \( 2A \sin kx \), the coefficient of the cosine function) depends on the location \( x \) of the element in the medium.

The maximum amplitude of an element of the medium has a minimum value of zero when \( x \) satisfies the condition \( \sin kx = 0 \), that is, when

\[ kx = \pi, 2\pi, 3\pi, \ldots \]

Because \( k = 2\pi/\lambda \), these values for \( kx \) give

\[ x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots = \frac{n\lambda}{2} \quad n = 0, 1, 2, 3, \ldots \]  

These points of zero amplitude are called **nodes**.
The element with the greatest possible displacement from equilibrium has an amplitude of $2A$, and we define this as the amplitude of the standing wave. The positions in the medium at which this maximum displacement occurs are called antinodes. The antinodes are located at positions for which the coordinate $x$ satisfies the condition $\sin kx = \pm 1$, that is, when

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

Thus, the positions of the antinodes are given by

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots = \frac{n\lambda}{4} \quad n = 1, 3, 5, \ldots \quad (18.5)$$

Position of antinodes

In examining Equations 18.4 and 18.5, we note the following important features of the locations of nodes and antinodes:

- The distance between adjacent antinodes is equal to $\lambda/2$.
- The distance between adjacent nodes is equal to $\lambda/2$.
- The distance between a node and an adjacent antinode is $\lambda/4$.

Wave patterns of the elements of the medium produced at various times by two waves traveling in opposite directions are shown in Figure 18.9. The blue and green curves are the wave patterns for the individual traveling waves, and the red curves are the wave patterns for the resultant standing wave. At $t = 0$ (Fig. 18.9a), the two traveling waves are in phase, giving a wave pattern in which each element of the medium is experiencing its maximum displacement from equilibrium. One quarter of a period later, at $t = T/4$ (Fig. 18.9b), the traveling waves have moved one quarter of a wavelength (one to the right and the other to the left). At this time, the traveling waves are out of phase, and each element of the medium is passing through the equilibrium position in its simple harmonic motion. The result is zero displacement for elements at all values of $x$—that is, the wave pattern is a straight line. At $t = T/2$ (Fig. 18.9c), the traveling waves are again in phase, producing a wave pattern that is inverted relative to the $t = 0$ pattern. In the standing wave, the elements of the medium alternate in time between the extremes shown in Figure 18.9a and c.

**Active Figure 18.9** Standing-wave patterns produced at various times by two waves of equal amplitude traveling in opposite directions. For the resultant wave $y$, the nodes (N) are points of zero displacement, and the antinodes (A) are points of maximum displacement.

At the Active Figures link at http://www.pse6.com, you can choose the wavelength of the waves and see the standing wave that results.
Quick Quiz 18.3 Consider a standing wave on a string as shown in Figure 18.9. Define the velocity of elements of the string as positive if they are moving upward in the figure. At the moment the string has the shape shown by the red curve in Figure 18.9a, the instantaneous velocity of elements along the string (a) is zero for all elements (b) is positive for all elements (c) is negative for all elements (d) varies with the position of the element.

Quick Quiz 18.4 Continuing with the scenario in Quick Quiz 18.3, at the moment the string has the shape shown by the red curve in Figure 18.9b, the instantaneous velocity of elements along the string (a) is zero for all elements (b) is positive for all elements (c) is negative for all elements (d) varies with the position of the element.

Example 18.2 Formation of a Standing Wave

Two waves traveling in opposite directions produce a standing wave. The individual wave functions are

\[ y_1 = (4.0 \text{ cm}) \sin(3.0x - 2.0t) \]
\[ y_2 = (4.0 \text{ cm}) \sin(3.0x + 2.0t) \]

where \(x\) and \(y\) are measured in centimeters.

**Solution** The standing wave is described by Equation 18.3; in this problem, we have \(A = 4.0 \text{ cm}, k = 3.0 \text{ rad/cm}, \text{ and } \omega = 2.0 \text{ rad/s}.

Thus, we obtain the amplitude of the simple harmonic motion of the element at the position \(x = 2.3 \text{ cm}\) by evaluating the coefficient of the cosine function at this position:

\[ y_{\text{max}} = (8.0 \text{ cm}) \sin(3.0x) \bigg|_{x=2.3} \]
\[ = (8.0 \text{ cm}) \sin(6.9 \text{ rad}) = 4.6 \text{ cm} \]

**Solution** With \(k = 2\pi/\lambda = 3.0 \text{ rad/cm}, \) we see that the wavelength is \(\lambda = (2\pi/3.0) \text{ cm}. \) Therefore, from Equation 18.4 we find that the nodes are located at

\[ x = n \frac{\lambda}{2} = n \left( \frac{\pi}{3} \right) \text{ cm} \]
\[ n = 0, 1, 2, 3, \ldots \]

and from Equation 18.5 we find that the antinodes are located at

\[ x = n \frac{\lambda}{4} = n \left( \frac{\pi}{6} \right) \text{ cm} \]
\[ n = 1, 3, 5, \ldots \]

**Solution** According to Equation 18.3, the maximum position of an element at an antinode is the amplitude of the standing wave, which is twice the amplitude of the individual traveling waves:

\[ y_{\text{max}} = 2A(\sin kx)_{\text{max}} = 2(4.0 \text{ cm})(\pm 1) = \pm 8.0 \text{ cm} \]

where we have used the fact that the maximum value of \(\sin kx\) is \(\pm 1.\) Let us check this result by evaluating the coefficient of our standing-wave function at the positions we found for the antinodes:

\[ y_{\text{max}} = (8.0 \text{ cm}) \sin 3.0x \bigg|_{x=\pm n(\pi/6)} \]
\[ = (8.0 \text{ cm}) \sin \left[ 3.0n \left( \frac{\pi}{6} \right) \text{ rad} \right] \]
\[ = (8.0 \text{ cm}) \sin \left[ n \left( \frac{\pi}{2} \right) \text{ rad} \right] = \pm 8.0 \text{ cm} \]

In evaluating this expression, we have used the fact that \(n\) is an odd integer; thus, the sine function is equal to \(\pm 1,\) depending on the value of \(n.\)

### 18.3 Standing Waves in a String Fixed at Both Ends

Consider a string of length \(L\) fixed at both ends, as shown in Figure 18.10. Standing waves are set up in the string by a continuous superposition of waves incident on and reflected from the ends. Note that there is a boundary condition for the waves on the
string. The ends of the string, because they are fixed, must necessarily have zero displacement and are, therefore, nodes by definition. The boundary condition results in the string having a number of natural patterns of oscillation, called normal modes, each of which has a characteristic frequency that is easily calculated. This situation in which only certain frequencies of oscillation are allowed is called quantization. Quantization is a common occurrence when waves are subject to boundary conditions and will be a central feature in our discussions of quantum physics in the extended version of this text.

Figure 18.11 shows one of the normal modes of oscillation of a string fixed at both ends. Except for the nodes, which are always stationary, all elements of the string oscillate vertically with the same frequency but with different amplitudes of simple harmonic motion. Figure 18.11 represents snapshots of the standing wave at various times over one half of a period. The red arrows show the velocities of various elements of the string at various times. As we found in Quick Quizzes 18.3 and 18.4,
all elements of the string have zero velocity at the extreme positions (Figs. 18.11a and 18.11e) and elements have varying velocities at other positions (Figs. 18.11b through 18.11d).

The normal modes of oscillation for the string can be described by imposing the requirements that the ends be nodes and that the nodes and antinodes be separated by one fourth of a wavelength. The first normal mode that is consistent with the boundary conditions, shown in Figure 18.10b, has nodes at its ends and one antinode in the middle. This is the longest-wavelength mode that is consistent with our requirements. This first normal mode occurs when the length of the string is half the wavelength $\lambda_1$, as indicated in Figure 18.10b, or $\lambda_1 = 2L$. The next normal mode (see Fig. 18.10c) of wavelength $\lambda_2$ occurs when the wavelength equals the length of the string, that is, when $\lambda_2 = L$. The third normal mode (see Fig. 18.10d) corresponds to the case in which $\lambda_3 = 2L/3$. In general, the wavelengths of the various normal modes for a string of length $L$ fixed at both ends are

$$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, \ldots \quad (18.6)$$

where the index $n$ refers to the $n$th normal mode of oscillation. These are the possible modes of oscillation for the string. The actual modes that are excited on a string are discussed shortly.

The natural frequencies associated with these modes are obtained from the relationship $f = v/\lambda$, where the wave speed $v$ is the same for all frequencies. Using Equation 18.6, we find that the natural frequencies $f_n$ of the normal modes are

$$f_n = \frac{v}{\lambda_n} = n \frac{v}{2L} \quad n = 1, 2, 3, \ldots \quad (18.7)$$

These natural frequencies are also called the quantized frequencies associated with the vibrating string fixed at both ends.

Because $v = \sqrt{T/\mu}$ (see Eq. 16.18), where $T$ is the tension in the string and $\mu$ is its linear mass density, we can also express the natural frequencies of a taut string as

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \quad n = 1, 2, 3, \ldots \quad (18.8)$$

The lowest frequency $f_1$, which corresponds to $n = 1$, is called either the fundamental or the fundamental frequency and is given by

$$f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \quad (18.9)$$

The frequencies of the remaining normal modes are integer multiples of the fundamental frequency. Frequencies of normal modes that exhibit an integer-multiple relationship such as this form a harmonic series, and the normal modes are called

Multiflash photographs of standing-wave patterns in a cord driven by a vibrator at its left end. The single-loop pattern represents the first normal mode (the fundamental), $n = 1$. The double-loop pattern represents the second normal mode ($n = 2$), and the triple-loop pattern represents the third normal mode ($n = 3$).
harmonics. The fundamental frequency $f_1$ is the frequency of the first harmonic; the frequency $f_2 = 2f_1$ is the frequency of the second harmonic; and the frequency $f_n = nf_1$ is the frequency of the $n$th harmonic. Other oscillating systems, such as a drumhead, exhibit normal modes, but the frequencies are not related as integer multiples of a fundamental. Thus, we do not use the term harmonic in association with these types of systems.

In obtaining Equation 18.6, we used a technique based on the separation distance between nodes and antinodes. We can obtain this equation in an alternative manner. Because we require that the string be fixed at $x = 0$ and $x = L$, the wave function $y(x, t)$ given by Equation 18.3 must be zero at these points for all times. That is, the boundary conditions require that $y(0, t) = 0$ and $y(L, t) = 0$ for all values of $t$. Because the standing wave is described by $y = 2A\sin(kx)\cos(\omega t)$, the first boundary condition, $y(0, t) = 0$, is automatically satisfied because $\sin kx = 0$ at $x = 0$. To meet the second boundary condition, $y(L, t) = 0$, we require that $\sin kL = 0$. This condition is satisfied when the angle $kL$ equals an integer multiple of $\pi$ rad. Therefore, the allowed values of $k$ are given by

$$k_nL = n\pi \quad n = 1, 2, 3, \ldots \quad (18.10)$$

Because $k_n = 2\pi/\lambda_n$, we find that

$$\left(\frac{2\pi}{\lambda_n}\right) L = n\pi \quad \text{or} \quad \lambda_n = \frac{2L}{n}$$

which is identical to Equation 18.6.

Let us examine further how these various harmonics are created in a string. If we wish to excite just a single harmonic, we must distort the string in such a way that its distorted shape corresponds to that of the desired harmonic. After being released, the string vibrates at the frequency of that harmonic. This maneuver is difficult to perform, however, and it is not how we excite a string of a musical instrument. If the string is distorted such that its distorted shape is not that of just one harmonic, the resulting vibration includes various harmonics. Such a distortion occurs in musical instruments when the string is plucked (as in a guitar), bowed (as in a cello), or struck (as in a piano). When the string is distorted into a nonsinusoidal shape, only waves that satisfy the boundary conditions can persist on the string. These are the harmonics.

The frequency of a string that defines the musical note that it plays is that of the fundamental. The frequency of the string can be varied by changing either the tension or the string’s length. For example, the tension in guitar and violin strings is varied by a screw adjustment mechanism or by tuning pegs located on the neck of the instrument. As the tension is increased, the frequency of the normal modes increases in accordance with Equation 18.8. Once the instrument is “tuned,” players vary the frequency by moving their fingers along the neck, thereby changing the length of the oscillating portion of the string. As the length is shortened, the frequency increases because, as Equation 18.8 specifies, the normal-mode frequencies are inversely proportional to string length.

Quick Quiz 18.5 When a standing wave is set up on a string fixed at both ends, (a) the number of nodes is equal to the number of antinodes (b) the wavelength is equal to the length of the string divided by an integer (c) the frequency is equal to the number of nodes times the fundamental frequency (d) the shape of the string at any time is symmetric about the midpoint of the string.

---

1 We exclude $n = 0$ because this value corresponds to the trivial case in which no wave exists ($k = 0$).
Example 18.3  Give Me a C Note!

Middle C on a piano has a fundamental frequency of 262 Hz, and the first A above middle C has a fundamental frequency of 440 Hz.

(A) Calculate the frequencies of the next two harmonics of the C string.

Solution Knowing that the frequencies of higher harmonics are integer multiples of the fundamental frequency \( f_1 = 262 \text{ Hz} \), we find that

\[
f_2 = 2f_1 = 524 \text{ Hz}
\]

\[
f_3 = 3f_1 = 786 \text{ Hz}
\]

(B) If the A and C strings have the same linear mass density \( \mu \) and length \( L \), determine the ratio of tensions in the two strings.

Solution Using Equation 18.8 for the two strings vibrating at their fundamental frequencies gives

\[
f_{1A} = \frac{1}{2L} \sqrt{\frac{T_A}{\mu}} \quad \text{and} \quad f_{1C} = \frac{1}{2L} \sqrt{\frac{T_C}{\mu}}
\]

Substituting the new frequency to find the shortened string length:

\[
L = n \frac{v}{2f_n} = (1) \frac{422 \text{ m/s}}{2(350 \text{ Hz})} = 0.603 \text{ m} = 60.3 \text{ cm}
\]

The difference between this length and the measured length of 64.0 cm is the distance from the fret to the neck end of the string, or 3.7 cm.

What If? What if we wish to play an F sharp, which we do by pressing down on the second fret from the neck in Figure 18.12? The frequency of F sharp is 370 Hz. Is this fret another 3.7 cm from the neck?

Answer If you inspect a guitar fingerboard, you will find that the frets are not equally spaced. They are far apart near the neck and close together near the opposite end. Consequently, from this observation, we would not expect the F sharp fret to be another 3.7 cm from the end.

Let us repeat the calculation of the string length, this time for the frequency of F sharp:

\[
L = n \frac{v}{2f_n} = (1) \frac{422 \text{ m/s}}{2(370 \text{ Hz})} = 0.571 \text{ m}
\]

This gives a distance of 0.640 m − 0.571 m = 0.069 m = 6.9 cm from the neck. Subtracting the distance from the neck to the first fret, the separation distance between the first and second frets is 6.9 cm − 3.7 cm = 3.2 cm.

Explore this situation at the Interactive Worked Example link at http://www.pse6.com.
Example 18.5 Changing String Vibration with Water

One end of a horizontal string is attached to a vibrating blade and the other end passes over a pulley as in Figure 18.13a. A sphere of mass 2.00 kg hangs on the end of the string. The string is vibrating in its second harmonic. A container of water is raised under the sphere so that the sphere is completely submerged. After this is done, the string vibrates in its fifth harmonic, as shown in Figure 18.13b. What is the radius of the sphere?

Solution To conceptualize the problem, imagine what happens when the sphere is immersed in the water. The buoyant force acts upward on the sphere, reducing the tension in the string. This altered wavelength results in the string vibrating in its fifth harmonic. When the sphere is immersed in water, the string vibrates in its second harmonic. Once the sphere is completely submerged, after this is done, the string vibrates in its second harmonic. A container of water is raised under the sphere so that the sphere is completely submerged. After this is done, the string vibrates in its fifth harmonic.

\[
\begin{align*}
\Sigma F &= T_1 - mg = 0 \\
T_1 &= mg = (2.00 \text{ kg})(9.80 \text{ m/s}^2) = 19.6 \text{ N}
\end{align*}
\]

where the subscript 1 is used to indicate initial variables before we immerse the sphere in water. Once the sphere is immersed in water, the tension in the string decreases to \( T_2 \). Applying Newton’s second law to the sphere again in this situation, we have

\[
T_2 + B - mg = 0 \\
B = mg - T_2
\]

The desired quantity, the radius of the sphere, will appear in the expression for the buoyant force \( B \). Before proceeding in this direction, however, we must evaluate \( T_2 \). We do this from the standing wave information. We write the equation for the frequency of a standing wave on a string (Equation 18.8) twice, once before we immerse the sphere and once after, and divide the equations:

\[
\begin{align*}
f &= \frac{n_1}{2L} \sqrt{\frac{T_1}{\mu}} \\
1 &= \frac{n_1}{n_2} \sqrt{\frac{T_1}{T_2}} \\
f &= \frac{n_2}{2L} \sqrt{\frac{T_2}{\mu}}
\end{align*}
\]

where the frequency \( f \) is the same in both cases, because it is determined by the vibrating blade. In addition, the linear mass density \( \mu \) and the length \( L \) of the vibrating portion of the string are the same in both cases. Solving for \( T_2 \), we have

\[
T_2 = \left(\frac{n_1}{n_2}\right)^2 T_1 = \left(\frac{9}{5}\right)^2 (19.6 \text{ N}) = 3.14 \text{ N}
\]

Substituting this into Equation (1), we can evaluate the buoyant force on the sphere:

\[
B = mg - T_2 = 19.6 \text{ N} - 3.14 \text{ N} = 16.5 \text{ N}
\]

Finally, expressing the buoyant force (Eq. 14.5) in terms of the radius of the sphere, we solve for the radius:

\[
B = \rho_{\text{water}} g V_{\text{sphere}} = \rho_{\text{water}} g \left(\frac{4}{3} \pi r^3\right)
\]

\[
r = \sqrt[3]{\frac{3B}{4\pi \rho_{\text{water}} g}} = \sqrt[3]{\frac{3(16.5 \text{ N})}{4\pi (1 \text{ 000 kg/m}^3)(9.80 \text{ m/s}^2)}}
\]

\[= 7.38 \times 10^{-2} \text{ m} = 7.38 \text{ cm}\]

To finalize this problem, note that only certain radii of the sphere will result in the string vibrating in a normal mode. This is because the speed of waves on the string must be changed to a value such that the length of the string is an integer multiple of half wavelengths. This is a feature of the quantization that we introduced earlier in this chapter—the sphere radii that cause the string to vibrate in a normal mode are quantized.

Figure 18.13 (Example 18.5) When the sphere hangs in air, the string vibrates in its second harmonic. When the sphere is immersed in water, the string vibrates in its fifth harmonic.
18.4 Resonance

We have seen that a system such as a taut string is capable of oscillating in one or more normal modes of oscillation. **If a periodic force is applied to such a system, the amplitude of the resulting motion is greatest when the frequency of the applied force is equal to one of the natural frequencies of the system.** We discussed this phenomenon, known as resonance, briefly in Section 15.7. Although a block–spring system or a simple pendulum has only one natural frequency, standing-wave systems have a whole set of natural frequencies, such as that given by Equation 18.7 for a string. Because an oscillating system exhibits a large amplitude when driven at any of its natural frequencies, these frequencies are often referred to as resonance frequencies.

Figure 18.14 shows the response of an oscillating system to various driving frequencies, where one of the resonance frequencies of the system is denoted by $f_0$. Note that the amplitude of oscillation of the system is greatest when the frequency of the driving force equals the resonance frequency. The maximum amplitude is limited by friction in the system. If a driving force does work on an oscillating system that is initially at rest, the input energy is used both to increase the amplitude of the oscillation and to overcome the friction force. Once maximum amplitude is reached, the work done by the driving force is used only to compensate for mechanical energy loss due to friction.

Examples of Resonance

A playground swing is a pendulum having a natural frequency that depends on its length. Whenever we use a series of regular impulses to push a child in a swing, the swing goes higher if the frequency of the periodic force equals the natural frequency of the swing. We can demonstrate a similar effect by suspending pendulums of different lengths from a horizontal support, as shown in Figure 18.15. If pendulum A is set into oscillation, the other pendulums begin to oscillate as a result of waves transmitted along the beam. However, pendulum C, the length of which is close to the length of A, oscillates with a much greater amplitude than pendulums B and D, the lengths of which are much different from that of pendulum A. Pendulum C moves the way it does because its natural frequency is nearly the same as the driving frequency associated with pendulum A.

Next, consider a taut string fixed at one end and connected at the opposite end to an oscillating blade, as illustrated in Figure 18.16. The fixed end is a node, and the end connected to the blade is very nearly a node because the amplitude of the blade’s motion is small compared with that of the elements of the string. As the blade oscillates, transverse waves sent down the string are reflected from the fixed end. As we learned in Section 18.3, the string has natural frequencies that are determined by its length, tension, and linear mass density (see Eq. 18.8). When the frequency of the blade equals one of the natural frequencies of the string, standing waves are produced and the string oscillates with a large amplitude. In this resonance case, the wave generated by the oscillating blade is in phase with the reflected wave, and the string absorbs energy from the blade. If the string is driven at a frequency that is not one of its natural frequencies, then the oscillations are of low amplitude and exhibit no stable pattern.

Once the amplitude of the standing-wave oscillations is a maximum, the mechanical energy delivered by the blade and absorbed by the system is transformed to internal energy because of the damping forces caused by friction in the system. If the applied frequency differs from one of the natural frequencies, energy is transferred to the string at first, but later the phase of the wave becomes such that it forces the blade to receive energy from the string, thereby reducing the energy in the string.

Resonance is very important in the excitation of musical instruments based on air columns. We shall discuss this application of resonance in Section 18.5.
Standing waves can be set up in a tube of air, such as that inside an organ pipe, as the result of interference between longitudinal sound waves traveling in opposite directions. The phase relationship between the incident wave and the wave reflected from one end of the pipe depends on whether that end is open or closed. This relationship is analogous to the phase relationships between incident and reflected transverse waves at the end of a string when the end is either fixed or free to move (see Figs. 16.14 and 16.15).

In a pipe closed at one end, the closed end is a displacement node because the wall at this end does not allow longitudinal motion of the air. As a result, at a closed end of a pipe, the reflected sound wave is 180° out of phase with the incident wave. Furthermore, because the pressure wave is 90° out of phase with the displacement wave (see Section 17.2), the closed end of an air column corresponds to a pressure antinode (that is, a point of maximum pressure variation).

The open end of an air column is approximately a displacement antinode and a pressure node. We can understand why no pressure variation occurs at an open end by noting that the end of the air column is open to the atmosphere; thus, the pressure at this end must remain constant at atmospheric pressure.

You may wonder how a sound wave can reflect from an open end, as there may not appear to be a change in the medium at this point. It is indeed true that the medium

Quick Quiz 18.6
A wine glass can be shattered through resonance by maintaining a certain frequency of a high-intensity sound wave. Figure 18.17a shows a side view of a wine glass vibrating in response to such a sound wave. Sketch the standing-wave pattern in the rim of the glass as seen from above. If an integral number of waves “fit” around the circumference of the vibrating rim, how many wavelengths fit around the rim in Figure 18.17a?

Figure 18.17 (Quick Quiz 18.6) (a) Standing-wave pattern in a vibrating wine glass. The glass shatters if the amplitude of vibration becomes too great. (b) A wine glass shattered by the amplified sound of a human voice.

18.5 Standing Waves in Air Columns

Standing waves can be set up in a tube of air, such as that inside an organ pipe, as the result of interference between longitudinal sound waves traveling in opposite directions. The phase relationship between the incident wave and the wave reflected from one end of the pipe depends on whether that end is open or closed. This relationship is analogous to the phase relationships between incident and reflected transverse waves at the end of a string when the end is either fixed or free to move (see Figs. 16.14 and 16.15).

In a pipe closed at one end, the closed end is a displacement node because the wall at this end does not allow longitudinal motion of the air. As a result, at a closed end of a pipe, the reflected sound wave is 180° out of phase with the incident wave. Furthermore, because the pressure wave is 90° out of phase with the displacement wave (see Section 17.2), the closed end of an air column corresponds to a pressure antinode (that is, a point of maximum pressure variation).

The open end of an air column is approximately a displacement antinode and a pressure node. We can understand why no pressure variation occurs at an open end by noting that the end of the air column is open to the atmosphere; thus, the pressure at this end must remain constant at atmospheric pressure.

You may wonder how a sound wave can reflect from an open end, as there may not appear to be a change in the medium at this point. It is indeed true that the medium

2 Strictly speaking, the open end of an air column is not exactly a displacement antinode. A compression reaching an open end does not reflect until it passes beyond the end. For a tube of circular cross section, an end correction equal to approximately 0.6R, where R is the tube’s radius, must be added to the length of the air column. Hence, the effective length of the air column is longer than the true length L. We ignore this end correction in this discussion.
through which the sound wave moves is air both inside and outside the pipe. However, sound is a pressure wave, and a compression region of the sound wave is constrained by the sides of the pipe as long as the region is inside the pipe. As the compression region exits at the open end of the pipe, the constraint of the pipe is removed and the compressed air is free to expand into the atmosphere. Thus, there is a change in the character of the medium between the inside of the pipe and the outside even though there is no change in the material of the medium. This change in character is sufficient to allow some reflection.

With the boundary conditions of nodes or antinodes at the ends of the air column, we have a set of normal modes of oscillation, as we do for the string fixed at both ends. Thus, the air column has quantized frequencies.

The first three normal modes of oscillation of a pipe open at both ends are shown in Figure 18.18a. Note that both ends are displacement antinodes (approximately). In the first normal mode, the standing wave extends between two adjacent antinodes, which is a distance of half a wavelength. Thus, the wavelength is twice the length of the pipe, and the fundamental frequency is \( f_1 = \frac{v}{2L} \). As Figure 18.18a shows, the frequencies of the higher harmonics are \( 2f_1, 3f_1, \ldots \). Thus, we can say that

In a pipe open at both ends, the natural frequencies of oscillation form a harmonic series that includes all integral multiples of the fundamental frequency.

---

**PITFALL PREVENTION**

18.3 Sound Waves in Air Are Longitudinal, not Transverse

Note that the standing longitudinal waves are drawn as transverse waves in Figure 18.18. This is because it is difficult to draw longitudinal displacements—they are in the same direction as the propagation. Thus, it is best to interpret the curves in Figure 18.18 as a graphical representation of the waves (our diagrams of string waves are pictorial representations), with the vertical axis representing horizontal displacement of the elements of the medium.

---

Figure 18.18 Motion of elements of air in standing longitudinal waves in a pipe, along with schematic representations of the waves. In the schematic representations, the structure at the left end has the purpose of exciting the air column into a normal mode. The hole in the upper edge of the column assures that the left end acts as an open end. The graphs represent the displacement amplitudes, not the pressure amplitudes. (a) In a pipe open at both ends, the harmonic series created consists of all integer multiples of the fundamental frequency: \( f_1, 2f_1, 3f_1, \ldots \). (b) In a pipe closed at one end and open at the other, the harmonic series created consists of only odd-integer multiples of the fundamental frequency: \( f_1, 3f_1, 5f_1, \ldots \).
Because all harmonics are present, and because the fundamental frequency is given by
the same expression as that for a string (see Eq. 18.7), we can express the natural fre-
quencies of oscillation as

\[ f_n = n \frac{v}{2L} \quad n = 1, 2, 3, \ldots \]  

(18.11)

Despite the similarity between Equations 18.7 and 18.11, you must remember that \( v \) in
Equation 18.7 is the speed of waves on the string, whereas \( v \) in Equation 18.11 is the
speed of sound in air.

If a pipe is closed at one end and open at the other, the closed end is a displace-
ment node (see Fig. 18.18b). In this case, the standing wave for the fundamental mode
extends from an antinode to the adjacent node, which is one fourth of a wavelength.
Hence, the wavelength for the first normal mode is \( 4L \), and the fundamental frequency
is \( f_1 = \frac{v}{4L} \). As Figure 18.18b shows, the higher-frequency waves that satisfy our condi-
tions are those that have a node at the closed end and an antinode at the open end;
this means that the higher harmonics have frequencies \( 3f_1, 5f_1, \ldots \).

In a pipe closed at one end, the natural frequencies of oscillation form a harmonic
series that includes only odd integral multiples of the fundamental frequency.

We express this result mathematically as

\[ f_n = n \frac{v}{4L} \quad n = 1, 3, 5, \ldots \]  

(18.12)

It is interesting to investigate what happens to the frequencies of instruments based
on air columns and strings during a concert as the temperature rises. The sound
emitted by a flute, for example, becomes sharp (increases in frequency) as it warms up
because the speed of sound increases in the increasingly warmer air inside the flute
(consider Eq. 18.11). The sound produced by a violin becomes flat (decreases in frequency)
as the strings thermally expand because the expansion causes their tension to
decrease (see Eq. 18.8).

Musical instruments based on air columns are generally excited by resonance. The
air column is presented with a sound wave that is rich in many frequencies. The air col-
umn then responds with a large-amplitude oscillation to the frequencies that match
the quantized frequencies in its set of harmonics. In many woodwind instruments, the
initial rich sound is provided by a vibrating reed. In the brasses, this excitation is pro-
vided by the sound coming from the vibration of the player’s lips. In a flute, the initial
excitation comes from blowing over an edge at the mouthpiece of the instrument. This
is similar to blowing across the opening of a bottle with a narrow neck. The sound of
the air rushing across the edge has many frequencies, including one that sets the air
cavity in the bottle into resonance.

Quick Quiz 18.7 A pipe open at both ends resonates at a fundamental
frequency \( f_{\text{open}} \). When one end is covered and the pipe is again made to resonate,
the fundamental frequency is \( f_{\text{closed}} \). Which of the following expressions describes
how these two resonant frequencies compare? (a) \( f_{\text{closed}} = f_{\text{open}} \)  (b) \( f_{\text{closed}} = \frac{1}{2} f_{\text{open}} \)  (c) \( f_{\text{closed}} = 2 f_{\text{open}} \)  (d) \( f_{\text{closed}} = \frac{3}{2} f_{\text{open}} \)

Quick Quiz 18.8 Balboa Park in San Diego has an outdoor organ. When
the air temperature increases, the fundamental frequency of one of the organ pipes
(a) stays the same (b) goes down (c) goes up (d) is impossible to determine.
**Example 18.6 Wind in a Culvert**

A section of drainage culvert 1.23 m in length makes a howling noise when the wind blows.

(A) Determine the frequencies of the first three harmonics of the culvert if it is cylindrical in shape and open at both ends. Take \( v = 343 \text{ m/s} \) as the speed of sound in air.

**Solution** The frequency of the first harmonic of a pipe open at both ends is

\[
 f_1 = \frac{v}{2L} = \frac{343 \text{ m/s}}{2(1.23 \text{ m})} = 139 \text{ Hz}
\]

Because both ends are open, all harmonics are present; thus,

\[
 f_2 = 2f_1 = 278 \text{ Hz} \quad \text{and} \quad f_3 = 3f_1 = 417 \text{ Hz}
\]

(B) What are the three lowest natural frequencies of the culvert if it is blocked at one end?

**Solution** The fundamental frequency of a pipe closed at one end is

\[
 f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(1.23 \text{ m})} = 69.7 \text{ Hz}
\]

In this case, only odd harmonics are present; hence, the next two harmonics have frequencies \( f_3 = 3f_1 = 209 \text{ Hz} \) and \( f_5 = 5f_1 = 349 \text{ Hz} \).

(C) For the culvert open at both ends, how many of the harmonics present fall within the normal human hearing range (20 to 20 000 Hz)?

**Solution** Because all harmonics are present for a pipe open at both ends, we can express the frequency of the highest harmonic heard as \( f_n = nf_1 \) where \( n \) is the number of harmonics that we can hear. For \( f_n = 20 000 \text{ Hz} \), we find that the number of harmonics present in the audible range is

\[
 n = \frac{20 000 \text{ Hz}}{139 \text{ Hz}} = 143
\]

Only the first few harmonics are of sufficient amplitude to be heard.

**Example 18.7 Measuring the Frequency of a Tuning Fork**

A simple apparatus for demonstrating resonance in an air column is depicted in Figure 18.19. A vertical pipe open at both ends is partially submerged in water, and a tuning fork vibrating at an unknown frequency is placed near the top of the pipe. The length \( L \) of the air column can be adjusted by moving the pipe vertically. The sound waves generated by the fork are reinforced when \( L \) corresponds to one of the resonance frequencies of the pipe.

For a certain pipe, the smallest value of \( L \) for which a peak occurs in the sound intensity is 9.00 cm. What are

(A) the frequency of the tuning fork

(B) the values of \( L \) for the next two resonance frequencies?

**Solution**

(A) Although the pipe is open at its lower end to allow the water to enter, the water’s surface acts like a wall at one end. Therefore, this setup can be modeled as an air column closed at one end, and so the fundamental frequency is given by \( f_1 = \frac{v}{4L} \). Taking \( v = 343 \text{ m/s} \) for the speed of sound in air and \( L = 0.090 \text{ m} \), we obtain

\[
 f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(0.090 \text{ m})} = 953 \text{ Hz}
\]

Because the tuning fork causes the air column to resonate at this frequency, this must also be the frequency of the tuning fork.

(B) Because the pipe is closed at one end, we know from Figure 18.18b that the wavelength of the fundamental mode is \( \lambda = 4L = 4(0.090 \text{ m}) = 0.360 \text{ m} \). Because the frequency of the tuning fork is constant, the next two normal modes (see Fig. 18.19b) correspond to lengths of

\[
 L = \frac{3\lambda}{4} = 0.270 \text{ m} \quad \text{and} \quad L = \frac{5\lambda}{4} = 0.450 \text{ m}
\]
18.6 Standing Waves in Rods and Membranes

Standing waves can also be set up in rods and membranes. A rod clamped in the middle and stroked parallel to the rod at one end oscillates, as depicted in Figure 18.20a. The oscillations of the elements of the rod are longitudinal, and so the broken lines in Figure 18.20 represent longitudinal displacements of various parts of the rod. For clarity, we have drawn them in the transverse direction, just as we did for air columns. The midpoint is a displacement node because it is fixed by the clamp, whereas the ends are displacement antinodes because they are free to oscillate. The oscillations in this setup are analogous to those in a pipe open at both ends. The broken lines in Figure 18.20a represent the first normal mode, for which the wavelength is $2L$ and the frequency is $f_1 = \frac{v}{2L}$, where $v$ is the speed of longitudinal waves in the rod. Other normal modes may be excited by clamping the rod at different points. For example, the second normal mode (Fig. 18.20b) is excited by clamping the rod a distance $L/4$ away from one end.

Musical instruments that depend on standing waves in rods include triangles, marimbas, xylophones, glockenspiels, chimes, and vibraphones. Other devices that make sounds from bars include music boxes and wind chimes.

Two-dimensional oscillations can be set up in a flexible membrane stretched over a circular hoop, such as that in a drumhead. As the membrane is struck at some point, waves that arrive at the fixed boundary are reflected many times. The resulting sound is not harmonic because the standing waves have frequencies that are not related by integer multiples. Without this relationship, the sound may be more correctly described as noise than as music. This is in contrast to the situation in wind and stringed instruments, which produce sounds that we describe as musical.

Some possible normal modes of oscillation for a two-dimensional circular membrane are shown in Figure 18.21. While nodes are points in one-dimensional standing

![Figure 18.20](image_url)

![Figure 18.21](image_url)

Elements of the medium moving out of the page at an instant of time.

Elements of the medium moving into the page at an instant of time.

Figure 18.21 Representation of some of the normal modes possible in a circular membrane fixed at its perimeter. The pair of numbers above each pattern corresponds to the number of radial nodes and the number of circular nodes. Below each pattern is a factor by which the frequency of the mode is larger than that of the 01 mode. The frequencies of oscillation do not form a harmonic series because these factors are not integers. In each diagram, elements of the membrane on either side of a nodal line move in opposite directions, as indicated by the colors. (Adapted from T. D. Rossing, The Science of Sound, 2nd ed, Reading, Massachusetts, Addison-Wesley Publishing Co., 1990)
waves on strings and in air columns, a two-dimensional oscillator has curves along which there is no displacement of the elements of the medium. The lowest normal mode, which has a frequency $f_1$, contains only one nodal curve; this curve runs around the outer edge of the membrane. The other possible normal modes show additional nodal curves that are circles and straight lines across the diameter of the membrane.

### 18.7 Beats: Interference in Time

The interference phenomena with which we have been dealing so far involve the superposition of two or more waves having the same frequency. Because the amplitude of the oscillation of elements of the medium varies with the position in space of the element, we refer to the phenomenon as spatial interference. Standing waves in strings and pipes are common examples of spatial interference.

We now consider another type of interference, one that results from the superposition of two waves having slightly different frequencies. In this case, when the two waves are observed at the point of superposition, they are periodically in and out of phase. That is, there is a temporal (time) alternation between constructive and destructive interference. As a consequence, we refer to this phenomenon as interference in time or *temporal interference*. For example, if two tuning forks of slightly different frequencies are struck, one hears a sound of periodically varying amplitude. This phenomenon is called **beating**: 

Beating is the periodic variation in amplitude at a given point due to the superposition of two waves having slightly different frequencies.

The number of amplitude maxima one hears per second, or the *beat frequency*, equals the difference in frequency between the two sources, as we shall show below. The maximum beat frequency that the human ear can detect is about 20 beats/s. When the beat frequency exceeds this value, the beats blend indistinguishably with the sounds producing them.

A piano tuner can use beats to tune a stringed instrument by “beating” a note against a reference tone of known frequency. The tuner can then adjust the string tension until the frequency of the sound it emits equals the frequency of the reference tone. The tuner does this by tightening or loosening the string until the beats produced by it and the reference source become too infrequent to notice.

Consider two sound waves of equal amplitude traveling through a medium with slightly different frequencies $f_1$ and $f_2$. We use equations similar to Equation 16.10 to represent the wave functions for these two waves at a point that we choose as $x = 0$:

\[
y_1 = A \cos \omega_1 t = A \cos 2\pi f_1 t \\
y_2 = A \cos \omega_2 t = A \cos 2\pi f_2 t
\]

Using the superposition principle, we find that the resultant wave function at this point is

\[
y = y_1 + y_2 = A(\cos 2\pi f_1 t + \cos 2\pi f_2 t)
\]

The trigonometric identity

\[
\cos a + \cos b = 2\cos \left( \frac{a - b}{2} \right) \cos \left( \frac{a + b}{2} \right)
\]

defines the beat frequency:

\[
2\pi (f_1 - f_2) = 2\pi \left( \frac{\Delta f}{2} \right)
\]

Here, $\Delta f = f_2 - f_1$ is the beat frequency. The maximum beat frequency that the human ear can detect is about 20 beats/s. When the beat frequency exceeds this value, the beats blend indistinguishably with the sounds producing them.

A piano tuner can use beats to tune a stringed instrument by “beating” a note against a reference tone of known frequency. The tuner can then adjust the string tension until the frequency of the sound it emits equals the frequency of the reference tone. The tuner does this by tightening or loosening the string until the beats produced by it and the reference source become too infrequent to notice.
allows us to write the expression for $y$ as

$$ y = 2A \cos 2\pi \left( f_1 - f_2 \right) t \cos 2\pi \left( \frac{f_1 + f_2}{2} \right) t $$

Graphs of the individual waves and the resultant wave are shown in Figure 18.22. From the factors in Equation 18.13, we see that the resultant sound for a listener standing at any given point has an effective frequency equal to the average frequency $(f_1 + f_2)/2$ and an amplitude given by the expression in the square brackets:

$$ A_{\text{resultant}} = 2A \cos 2\pi \left( \frac{f_1 - f_2}{2} \right) t $$

That is, the **amplitude and therefore the intensity of the resultant sound vary in time**. The broken blue line in Figure 18.22b is a graphical representation of Equation 18.14 and is a sine wave varying with frequency $(f_1 - f_2)/2$.

Note that a maximum in the amplitude of the resultant sound wave is detected whenever

$$ \cos 2\pi \left( \frac{f_1 - f_2}{2} \right) t = \pm 1 $$

This means there are **two** maxima in each period of the resultant wave. Because the amplitude varies with frequency as $(f_1 - f_2)/2$, the number of beats per second, or the beat frequency $f_{\text{beat}}$, is twice this value. That is,

$$ f_{\text{beat}} = \left| f_1 - f_2 \right| $$

For instance, if one tuning fork vibrates at 438 Hz and a second one vibrates at 442 Hz, the resultant sound wave of the combination has a frequency of 440 Hz (the musical note A) and a beat frequency of 4 Hz. A listener would hear a 440-Hz sound wave go through an intensity maximum four times every second.

**Quick Quiz 18.9** You are tuning a guitar by comparing the sound of the string with that of a standard tuning fork. You notice a beat frequency of 5 Hz when both sounds are present. You tighten the guitar string and the beat frequency rises to 8 Hz. In order to tune the string exactly to the tuning fork, you should (a) continue to tighten the string (b) loosen the string (c) impossible to determine.
Example 18.8 The Mistuned Piano Strings

Two identical piano strings of length 0.750 m are each tuned exactly to 440 Hz. The tension in one of the strings is then increased by 1.0%. If they are now struck, what is the beat frequency between the fundamentals of the two strings?

Solution We find the ratio of frequencies if the tension in one string is 1.0% larger than the other:

\[
\frac{f_2}{f_1} = \frac{\left(\frac{v_2}{2L}\right)}{\left(\frac{v_1}{2L}\right)} = \frac{v_2}{v_1} = \frac{\sqrt{T_2/\mu}}{\sqrt{T_1/\mu}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{1.010T_1}{T_1}} = 1.005
\]

Thus, the frequency of the tightened string is

\[f_2 = 1.005f_1 = 1.005(440 \text{ Hz}) = 442 \text{ Hz}\]

and the beat frequency is

\[f_{\text{beat}} = 442 \text{ Hz} - 440 \text{ Hz} = 2 \text{ Hz}.
\]

18.8 Nonsinusoidal Wave Patterns

The sound wave patterns produced by the majority of musical instruments are nonsinusoidal. Characteristic patterns produced by a tuning fork, a flute, and a clarinet, each playing the same note, are shown in Figure 18.23. Each instrument has its own characteristic pattern. Note, however, that despite the differences in the patterns, each pattern is periodic. This point is important for our analysis of these waves.

It is relatively easy to distinguish the sounds coming from a violin and a saxophone even when they are both playing the same note. On the other hand, an individual untrained in music may have difficulty distinguishing a note played on a clarinet from the same note played on an oboe. We can use the pattern of the sound waves from various sources to explain these effects.

This is in contrast to a musical instrument that makes a noise, such as the drum, in which the combination of frequencies do not form a harmonic series. When frequencies that are integer multiples of a fundamental frequency are combined, the result is a musical sound. A listener can assign a pitch to the sound, based on the fundamental frequency. Pitch is a psychological reaction to a sound that allows the listener to place the sound on a scale of low to high (bass to treble). Combinations of frequencies that are not integer multiples of a fundamental result in a noise, rather than a musical sound. It is much harder for a listener to assign a pitch to a noise than to a musical sound.

The sound waves produced by a musical instrument are the result of the superposition of various harmonics. This superposition results in the corresponding richness of musical tones. The human perceptive response associated with various mixtures of harmonics is the quality or timbre of the sound. For instance, the sound of the trumpet is perceived to have a “brassy” quality (that is, we have learned to associate the adjective brassy with that sound); this quality enables us to distinguish the sound of the trumpet from that of the saxophone, whose quality is perceived as “reedy.” The clarinet and oboe, however, both contain air columns excited by reeds; because of this similarity, it is more difficult for the ear to distinguish them on the basis of their sound quality.

The problem of analyzing nonsinusoidal wave patterns appears at first sight to be a formidable task. However, if the wave pattern is periodic, it can be represented as closely as desired by the combination of a sufficiently large number of sinusoidal waves that form a harmonic series. In fact, we can represent any periodic function as a series of sine and cosine terms by using a mathematical technique based on Fourier’s theorem. The corresponding sum of terms that represents the periodic wave pattern

---

3 Developed by Jean Baptiste Joseph Fourier (1786–1830).
is called a Fourier series. Let \( y(t) \) be any function that is periodic in time with period \( T \), such that \( y(t + T) = y(t) \). Fourier’s theorem states that this function can be written as

\[
y(t) = \sum_n \left( A_n \sin 2\pi f_n t + B_n \cos 2\pi f_n t \right)
\]

(18.16) Fourier’s theorem

where the lowest frequency is \( f_1 = 1/T \). The higher frequencies are integer multiples of the fundamental, \( f_n = nf_1 \), and the coefficients \( A_n \) and \( B_n \) represent the amplitudes of the various waves. Figure 18.24 represents a harmonic analysis of the wave patterns shown in Figure 18.23. Note that a struck tuning fork produces only one harmonic (the first), whereas the flute and clarinet produce the first harmonic and many higher ones.

Note the variation in relative intensity of the various harmonics for the flute and the clarinet. In general, any musical sound consists of a fundamental frequency \( f \) plus other frequencies that are integer multiples of \( f \), all having different intensities.

### PITFALL PREVENTION

18.4 Pitch vs. Frequency

Do not confuse the term pitch with frequency. Frequency is the physical measurement of the number of oscillations per second. Pitch is a psychological reaction to sound that enables a person to place the sound on a scale from high to low, or from treble to bass. Thus, frequency is the stimulus and pitch is the response. Although pitch is related mostly (but not completely) to frequency, they are not the same. A phrase such as “the pitch of the sound” is incorrect because pitch is not a physical property of the sound.
We have discussed the analysis of a wave pattern using Fourier’s theorem. The analysis involves determining the coefficients of the harmonics in Equation 18.16 from a knowledge of the wave pattern. The reverse process, called Fourier synthesis, can also be performed. In this process, the various harmonics are added together to form a resultant wave pattern. As an example of Fourier synthesis, consider the building of a square wave, as shown in Figure 18.25. The symmetry of the square wave results in only odd multiples of the fundamental frequency combining in its synthesis. In Figure 18.25a, the orange curve shows the combination of $f$ and $3f$. In Figure 18.25b, we have added $5f$ to the combination and obtained the green curve. Notice how the general shape of the square wave is approximated, even though the upper and lower portions are not flat as they should be.

Figure 18.25c shows the result of adding odd frequencies up to $9f$. This approximation (purple curve) to the square wave is better than the approximations in parts a and b. To approximate the square wave as closely as possible, we would need to add all odd multiples of the fundamental frequency, up to infinite frequency.

Using modern technology, we can generate musical sounds electronically by mixing different amplitudes of any number of harmonics. These widely used electronic music synthesizers are capable of producing an infinite variety of musical tones.
The **superposition principle** specifies that when two or more waves move through a medium, the value of the resultant wave function equals the algebraic sum of the values of the individual wave functions.

When two traveling waves having equal amplitudes and frequencies superimpose, the resultant wave has an amplitude that depends on the phase angle $\phi$ between the two waves. **Constructive interference** occurs when the two waves are in phase, corresponding to $\phi = 0, 2\pi, 4\pi, \ldots$ rad. **Destructive interference** occurs when the two waves are $180^\circ$ out of phase, corresponding to $\phi = \pi, 3\pi, 5\pi, \ldots$ rad.

**Standing waves** are formed from the superposition of two sinusoidal waves having the same frequency, amplitude, and wavelength but traveling in opposite directions. The resultant standing wave is described by the wave function

$$y = (2A \sin kx) \cos \omega t \quad (18.3)$$

Hence, the amplitude of the standing wave is $2A$, and the amplitude of the simple harmonic motion of any particle of the medium varies according to its position as $2A \sin kx$.

The points of zero amplitude (called **nodes**) occur at $x = n\lambda/2$ ($n = 0, 1, 2, 5, \ldots$). The maximum amplitude points (called **antinodes**) occur at $x = n\lambda/4$ ($n = 1, 3, 5, \ldots$). Adjacent antinodes are separated by a distance $\lambda/2$. Adjacent nodes also are separated by a distance $\lambda/2$.

The natural frequencies of vibration of a taut string of length $L$ and fixed at both ends are quantized and are given by

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \quad n = 1, 2, 3, \ldots \quad (18.8)$$

where $T$ is the tension in the string and $\mu$ is its linear mass density. The natural frequencies of vibration $f_1, 2f_1, 3f_1, \ldots$ form a **harmonic series**.

An oscillating system is in **resonance** with some driving force whenever the frequency of the driving force matches one of the natural frequencies of the system. When the system is resonating, it responds by oscillating with a relatively large amplitude.

Standing waves can be produced in a column of air inside a pipe. If the pipe is open at both ends, all harmonics are present and the natural frequencies of oscillation are

$$f_n = n\frac{v}{2L} \quad n = 1, 2, 3, \ldots \quad (18.11)$$

If the pipe is open at one end and closed at the other, only the odd harmonics are present, and the natural frequencies of oscillation are

$$f_n = n\frac{v}{4L} \quad n = 1, 3, 5, \ldots \quad (18.12)$$

The phenomenon of **beating** is the periodic variation in intensity at a given point due to the superposition of two waves having slightly different frequencies.

**QUESTIONS**

1. Does the phenomenon of wave interference apply only to sinusoidal waves?
2. As oppositely moving pulses of the same shape (one upward, one downward) on a string pass through each other, there is one instant at which the string shows no displacement from the equilibrium position at any point. Has the energy carried by the pulses disappeared at this instant of time? If not, where is it?
3. Can two pulses traveling in opposite directions on the same string reflect from each other? Explain.
4. When two waves interfere, can the amplitude of the resultant wave be greater than either of the two original waves? Under what conditions?
5. For certain positions of the movable section shown in Figure 18.5, no sound is detected at the receiver—a situation
corresponding to destructive interference. This suggests that energy is somehow lost. What happens to the energy transmitted by the speaker?

6. When two waves interfere constructively or destructively, is there any gain or loss in energy? Explain.

7. A standing wave is set up on a string, as shown in Figure 18.10. Explain why no energy is transmitted along the string.

8. What limits the amplitude of motion of a real vibrating system that is driven at one of its resonant frequencies?

9. Explain why your voice seems to sound better than usual when you sing in the shower.

10. What is the purpose of the slide on a trombone or of the valves on a trumpet?

11. Explain why all harmonics are present in an organ pipe open at both ends, but only odd harmonics are present in a pipe closed at one end.

12. Explain how a musical instrument such as a piano may be tuned by using the phenomenon of beats.

13. To keep animals away from their cars, some people mount short, thin pipes on the fenders. The pipes give out a high-pitched whine when the cars are moving. How do they create the sound?

14. When a bell is rung, standing waves are set up around the bell’s circumference. What boundary conditions must be satisfied by the resonant wavelengths? How does a crack in the bell, such as in the Liberty Bell, affect the satisfying of the boundary conditions and the sound emanating from the bell?

15. An archer shoots an arrow from a bow. Does the string of the bow exhibit standing waves after the arrow leaves? If so, and if the bow is perfectly symmetric so that the arrow leaves from the center of the string, what harmonics are excited?

16. Despite a reasonably steady hand, a person often spills his coffee when carrying it to his seat. Discuss resonance as a possible cause of this difficulty, and devise a means for solving the problem.

17. An airplane mechanic notices that the sound from a twin-engine aircraft rapidly varies in loudness when both engines are running. What could be causing this variation from loud to soft?

18. When the base of a vibrating tuning fork is placed against a chalkboard, the sound that it emits becomes louder. This is because the vibrations of the tuning fork are transmitted to the chalkboard. Because it has a larger area than the tuning fork, the vibrating chalkboard sets more air into vibration. Thus, the chalkboard is a better radiator of sound than the tuning fork. How does this affect the length of time during which the fork vibrates? Does this agree with the principle of conservation of energy?

19. If you wet your finger and lightly run it around the rim of a fine wineglass, a high-frequency sound is heard. Why? How could you produce various musical notes with a set of wineglasses, each of which contains a different amount of water?

20. If you inhale helium from a balloon and do your best to speak normally, your voice will have a comical quacky quality. Explain why this “Donald Duck effect” happens. Caution: Helium is an asphyxiating gas and asphyxiation can cause panic. Helium can contain poisonous contaminants.

21. You have a standard tuning fork whose frequency is 262 Hz and a second tuning fork with an unknown frequency. When you tap both of them on the heel of one of your sneakers, you hear beats with a frequency of 4 per second. Thoughtfully chewing your gum, you wonder whether the unknown frequency is 258 Hz or 266 Hz. How can you decide?

---

### Problems

**Section 18.1 Superposition and Interference**

1. Two waves in one string are described by the wave functions

   \[ y_1 = 3.0 \cos(4.0x - 1.6t) \]

   and

   \[ y_2 = 4.0 \sin(5.0x - 2.0t) \]

   where \( y \) and \( x \) are in centimeters and \( t \) is in seconds. Find the superposition of the waves \( y_1 + y_2 \) at the points (a) \( x = 1.00, t = 1.00 \), (b) \( x = 1.00, t = 0.500 \), and (c) \( x = 0.500, t = 0 \). (Remember that the arguments of the trigonometric functions are in radians.)

2. Two pulses A and B are moving in opposite directions along a taut string with a speed of 2.00 cm/s. The amplitude of A is twice the amplitude of B. The pulses are shown in Figure P18.2 at \( t = 0 \). Sketch the shape of the string at \( t = 1, 1.5, 2, 2.5, \) and 3 s.

---

**Figure P18.2**
5. Two pulses traveling on the same string are described by

\[ y_1 = \frac{5}{(3x - 4t)^2 + 2} \quad \text{and} \quad y_2 = \frac{-5}{(3x + 4t - 6)^2 + 2} \]

(a) In which direction does each pulse travel? (b) At what time do the two cancel everywhere? (c) At what point do the two pulses always cancel?

4. Two waves are traveling in the same direction along a stretched string. The waves are 90.0° out of phase. Each wave has an amplitude of 4.00 cm. Find the amplitude of the resultant wave.

5. Two traveling sinusoidal waves are described by the wave functions

\[ y_1 = (5.00 \text{ m}) \sin[\pi(4.00x - 1200t)] \]

and

\[ y_2 = (5.00 \text{ m}) \sin[\pi(4.00x - 1200t - 0.250)] \]

where \( x, y_1, \) and \( y_2 \) are in meters and \( t \) is in seconds. (a) What is the amplitude of the resultant wave? (b) What is the frequency of the resultant wave?

6. Two identical sinusoidal waves with wavelengths of 3.00 m travel in the same direction at a speed of 2.00 m/s. The second wave originates from the same point as the first, but at a later time. Determine the minimum possible time interval between the starting moments of the two waves if the amplitude of the resultant wave is the same as that of each of the two initial waves.

7. Review problem. A series of pulses, each of amplitude 0.150 m, is sent down a string that is attached to a post at one end. The pulses are reflected at the post and travel back along the string without loss of amplitude. What is the net displacement at a point on the string where two pulses are crossing? (a) if the string is rigidly attached to the post? (b) if the end at which reflection occurs is free to slide up and down?

8. Two loudspeakers are placed on a wall 2.00 m apart. A listener stands 3.00 m from the wall directly in front of one of the speakers. A single oscillator is driving the speakers at a frequency of 300 Hz. (a) What is the phase difference between the two waves when they reach the observer? (b) What if? What is the frequency closest to 300 Hz to which the oscillator may be adjusted such that the observer hears minimal sound?

9. Two speakers are driven by the same oscillator whose frequency is 200 Hz. They are located on a vertical pole a distance of 4.00 m from each other. A man walks straight toward the lower speaker in a direction perpendicular to the pole as shown in Figure P18.9. (a) How many times will he hear a minimum in sound intensity? (b) How far is he from the pole at these moments? Let \( v \) represent the speed of sound, and assume that the ground does not reflect sound.

10. Two speakers are driven by the same oscillator whose frequency is \( f \). They are located a distance \( d \) from each other on a vertical pole. A man walks straight toward the lower speaker in a direction perpendicular to the pole, as shown in Figure P18.9. (a) How many times will he hear a minimum in sound intensity? (b) How far is he from the pole at these moments? Let \( v \) represent the speed of sound, and assume that the ground does not reflect sound.
mine the relationship between $x$ and $y$ (the coordinates of the receiver) that causes the receiver to record a minimum in sound intensity. Take the speed of sound to be 344 m/s.

Section 18.2 Standing Waves

13. Two sinusoidal waves traveling in opposite directions interfere to produce a standing wave with the wave function

$$y = (1.50 \text{ m}) \sin(0.400x) \cos(200t)$$

where $x$ is in meters and $t$ is in seconds. Determine the wavelength, frequency, and speed of the interfering waves.

14. Two waves in a long string have wave functions given by

$$y_1 = (0.015 \text{ m}) \cos \left( \frac{x}{2} - 40t \right)$$

and

$$y_2 = (0.015 \text{ m}) \cos \left( \frac{x}{2} + 40t \right)$$

where $y_1$, $y_2$, and $x$ are in meters and $t$ is in seconds. (a) Determine the positions of the nodes of the resulting standing wave. (b) What is the maximum transverse position of an element of the string at the position $x = 0.400 \text{ m}$?

15. Two speakers are driven in phase by a common oscillator at 800 Hz and face each other at a distance of 1.25 m. Locate the points along a line joining the two speakers where relative minima of sound pressure amplitude would be expected. (Use $v = 345 \text{ m/s}$.)

16. Verify by direct substitution that the wave function for a standing wave given in Equation 18.3,

$$y = 2A \sin kx \cos \omega t$$

is a solution of the general linear wave equation, Equation 16.27:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

17. Two sinusoidal waves combining in a medium are described by the wave functions

$$y_1 = (3.0 \text{ cm}) \sin \pi(x + 0.60t)$$

and

$$y_2 = (3.0 \text{ cm}) \sin \pi(x - 0.60t)$$

where $x$ is in centimeters and $t$ is in seconds. Determine the maximum transverse position of an element of the medium at (a) $x = 0.250 \text{ cm}$, (b) $x = 0.500 \text{ cm}$, and (c) $x = 1.50 \text{ cm}$. (d) Find the three smallest values of $x$ corresponding to antinodes.

18. Two waves that set up a standing wave in a long string are given by the wave functions

$$y_1 = A \sin(kx - \omega t + \phi)$$

and

$$y_2 = A \sin(kx + \omega t)$$

Show (a) that the addition of the arbitrary phase constant $\phi$ changes only the position of the nodes and, in particular, (b) that the distance between nodes is still one half the wavelength.

Section 18.3 Standing Waves in a String Fixed at Both Ends

19. Find the fundamental frequency and the next three frequencies that could cause standing-wave patterns on a string that is 30.0 m long, has a mass per length of $9.00 \times 10^{-3} \text{ kg/m}$, and is stretched to a tension of 20.0 N.

20. A string with a mass of 8.00 g and a length of 5.00 m has one end attached to a wall; the other end is draped over a pulley and attached to a hanging object with a mass of 4.00 kg. If the string is plucked, what is the fundamental frequency of vibration?

21. In the arrangement shown in Figure P18.21, an object can be hung from a string (with linear mass density $\mu = 0.002 \text{ 00 kg/m}$) that passes over a light pulley. The string is connected to a vibrator (of constant frequency $f$), and the length of the string between point $P$ and the pulley is $L = 2.00 \text{ m}$. When the mass $m$ of the object is either 16.0 kg or 25.0 kg, standing waves are observed; however, no standing waves are observed with any mass between these values. (a) What is the frequency of the vibrator? (Note: The greater the tension in the string, the smaller the number of nodes in the standing wave.) (b) What is the largest object mass for which standing waves could be observed?

22. A vibrator, pulley, and hanging object are arranged as in Figure P18.21, with a compound string, consisting of two strings of different masses and lengths fastened together end-to-end. The first string, which has a mass of 1.56 g and a length of 65.8 cm, runs from the vibrator to the junction of the two strings. The second string runs from the junction over the pulley to the suspended 6.93-kg object. The mass and length of the string from the junction to the pulley are, respectively, 6.75 g and 95.0 cm. (a) Find the lowest frequency for which standing waves are observed in both strings, with a node at the junction. The standing wave patterns in the two strings may have different numbers of nodes. (b) What is the total number of nodes observed along the compound string at this frequency, excluding the nodes at the vibrator and the pulley?

23. Example 18.4 tells you that the adjacent notes E, F, and F-sharp can be assigned frequencies of 330 Hz, 350 Hz, and 370 Hz. You might not guess how the pattern continues. The next notes, G, G-sharp, and A, have frequencies of 392 Hz, 416 Hz, and 440 Hz. On the equally tempered or chromatic scale used in Western music, the frequency of each higher note is obtained by multiplying the previous frequency by $\sqrt[12]{2}$. A standard guitar has strings 64.0 cm long and nineteen frets. In Example 18.4, we found the
spacings of the first two frets. Calculate the distance between the last two frets.

24. The top string of a guitar has a fundamental frequency of 330 Hz when it is allowed to vibrate as a whole, along all of its 64.0-cm length from the neck to the bridge. A fret is provided for limiting vibration to just the lower two-thirds of the string. (a) If the string is pressed down at this fret and plucked, what is the new fundamental frequency? (b) What If? The guitarist can play a "natural harmonic" by gently touching the string at the location of this fret and plucking the string at about one sixth of the way along its length from the bridge. What frequency will be heard then?

25. A string of length \( L \), mass per unit length \( \mu \), and tension \( T \) is vibrating at its fundamental frequency. What effect will the following have on the fundamental frequency? (a) The length of the string is doubled, with all other factors held constant. (b) The mass per unit length is doubled, with all other factors held constant. (c) The tension is doubled, with all other factors held constant.

26. A 60.000-cm guitar string under a tension of 50.000 N has a mass per unit length of 0.100 00 g/cm. What is the highest resonant frequency that can be heard by a person capable of hearing frequencies up to 20 000 Hz?

27. A cello A-string vibrates in its first normal mode with a frequency of 220 Hz. The vibrating segment is 70.0 cm long and has a mass of 1.20 g. (a) Find the tension in the string. (b) Determine the frequency of vibration when the string vibrates in three segments.

28. A violin string has a length of 0.350 m and is tuned to concert G, with \( f_G = 392 \) Hz. Where must the violinist place her finger to play concert A, with \( f_A = 440 \) Hz? If this position is to remain correct to half the width of a finger (that is, to within 0.600 cm), what is the maximum allowable percentage change in the string tension?

29. Review problem. A sphere of mass \( M \) is supported by a string that passes over a light horizontal rod of length \( L \) (Fig. P18.29). Given that the angle is \( \theta \) and that \( f \) represents the fundamental frequency of standing waves in the portion of the string above the rod, determine the mass of this portion of the string.

30. Review problem. A copper cylinder hangs at the bottom of a steel wire of negligible mass. The top end of the wire is fixed. When the wire is struck, it emits sound with a fundamental frequency of 300 Hz. If the copper cylinder is then submerged in water so that half its volume is below the water line, determine the new fundamental frequency.

31. A standing-wave pattern is observed in a thin wire with a length of 3.00 m. The equation of the wave is

\[ y = (0.002 \, \text{m}) \sin(\pi x) \cos(100\pi t) \]

where \( x \) is in meters and \( t \) is in seconds. (a) How many loops does this pattern exhibit? (b) What is the fundamental frequency of vibration of the wire? (c) What If? If the original frequency is held constant and the tension in the wire is increased by a factor of 9, how many loops are present in the new pattern?

Section 18.4 Resonance

32. The chains suspending a child’s swing are 2.00 m long. At what frequency should a big brother push to make the child swing with largest amplitude?

33. An earthquake can produce a seiche in a lake, in which the water sloshes back and forth from end to end with remarkably large amplitude and long period. Consider a seiche produced in a rectangular farm pond, as in the cross-sectional view of Figure P18.33. (The figure is not drawn to scale.) Suppose that the pond is 9.15 m long and of uniform width and depth. You measure that a pulse produced at one end reaches the other end in 2.50 s. (a) What is the wave speed? (b) To produce the seiche, several people stand on the bank at one end and paddle together with snow shovels, moving them in simple harmonic motion. What should be the frequency of this motion?

![Figure P18.33](image-url)

34. The Bay of Fundy, Nova Scotia, has the highest tides in the world, as suggested in the photographs on page 452. Assume that in mid-ocean and at the mouth of the bay, the Moon’s gravity gradient and the Earth’s rotation make the water surface oscillate with an amplitude of a few centimeters and a period of 12 h 24 min. At the head of the bay, the amplitude is several meters. Argue for or against the
proposition that the tide is amplified by standing-wave resonance. Assume the bay has a length of 210 km and a uniform depth of 36.1 m. The speed of long-wavelength water waves is given by \( \sqrt{gd} \), where \( d \) is the water’s depth.

35. Standing-wave vibrations are set up in a crystal goblet with four nodes and four antinodes equally spaced around the 20.0-cm circumference of its rim. If transverse waves move around the glass at 900 m/s, an opera singer would have to produce a high harmonic with what frequency to shatter the glass with a resonant vibration?

Section 18.5 Standing Waves in Air Columns

Note: Unless otherwise specified, assume that the speed of sound in air is 343 m/s at 20°C, and is described by

\[ v = (331 \text{ m/s}) \sqrt{1 + \frac{T_C}{273}} \]

at any Celsius temperature \( T_C \).

36. The overall length of a piccolo is 32.0 cm. The resonating air column vibrates as in a pipe open at both ends. (a) Find the frequency of the lowest note that a piccolo can play, assuming that the speed of sound in air is 340 m/s. (b) Opening holes in the side effectively shortens the length of the resonant column. If the highest note a piccolo can sound is 4,000 Hz, find the distance between adjacent antinodes for this mode of vibration.

37. Calculate the length of a pipe that has a fundamental frequency of 240 Hz if the pipe is (a) closed at one end and (b) open at both ends.

38. The fundamental frequency of an open organ pipe corresponds to middle C (261.6 Hz on the chromatic musical scale). The third resonance of a closed organ pipe has the same frequency. What are the lengths of the two pipes?

39. The windpipe of one typical whooping crane is 5.00 ft long. What is the fundamental resonant frequency of the bird’s trachea, modeled as a narrow pipe closed at one end? Assume a temperature of 37°C.

40. Do not stick anything into your ear! Estimate the length of your ear canal, from its opening at the external ear to the eardrum. If you regard the canal as a narrow tube that is open at one end and closed at the other, at approximately what fundamental frequency would you expect your hearing to be most sensitive? Explain why you can hear especially soft sounds just around this frequency.

41. A shower stall measures 86.0 cm \( \times \) 86.0 cm \( \times \) 210 cm. If you were singing in this shower, which frequencies would sound the richest (because of resonance)? Assume that the stall acts as a pipe closed at both ends, with nodes at opposite sides. Assume that the voices of various singers range from 130 Hz to 2,000 Hz. Let the speed of sound in the hot shower stall be 355 m/s.

42. As shown in Figure P18.42, water is pumped into a tall vertical cylinder at a volume flow rate \( R \). The radius of the cylinder is \( r \) and at the open top of the cylinder a tuning fork is vibrating with a frequency \( f \). As the water rises, how much time elapses between successive resonances?

43. If two adjacent natural frequencies of an organ pipe are determined to be 550 Hz and 650 Hz, calculate the fundamental frequency and length of this pipe. (Use \( v = 340 \text{ m/s} \).)

44. A glass tube (open at both ends) of length \( L \) is positioned near an audio speaker of frequency \( f = 680 \text{ Hz} \). For what values of \( L \) will the tube resonate with the speaker?

45. An air column in a glass tube is open at one end and closed at the other by a movable piston. The air in the tube is warmed above room temperature, and a 384-Hz tuning fork is held at the open end. Resonance is heard when the piston is 22.8 cm from the open end and again when it is 68.3 cm from the open end. (a) What speed of sound is implied by these data? (b) How far from the open end will the piston be when the next resonance is heard?

46. A tuning fork with a frequency of 512 Hz is placed near the top of the pipe shown in Figure 18.19a. The water level is lowered so that the length \( L \) slowly increases from an initial value of 20.0 cm. Determine the next two values of \( L \) that correspond to resonant modes.

47. When an open metal pipe is cut into two pieces, the lowest resonance frequency for the air column in one piece is 256 Hz and that for the other is 440 Hz. (a) What resonant frequency would have been produced by the original length of pipe? (b) How long was the original pipe?

48. With a particular fingering, a flute plays a note with frequency 880 Hz at 20.0°C. The flute is open at both ends. (a) Find the air column length. (b) Find the frequency it produces at the beginning of the half-time performance at a late-season American football game, when the ambient temperature is -5.00°C and the musician has not had a chance to warm up the flute.
Section 18.6 Standing Waves in Rods and Membranes

49. An aluminum rod 1.60 m long is held at its center. It is stroked with a rosin-coated cloth to set up a longitudinal vibration. The speed of sound in a thin rod of aluminum is 5 100 m/s. (a) What is the fundamental frequency of the waves established in the rod? (b) What harmonics are set up in the rod held in this manner? (c) What If? What would be the fundamental frequency if the rod were made of copper, in which the speed of sound is 3 560 m/s?

50. An aluminum rod is clamped one quarter of the way along its length and set into longitudinal vibration by a variable-frequency driving source. The lowest frequency that produces resonance is 4 400 Hz. The speed of sound in an aluminum rod is 5 100 m/s. Find the length of the rod.

Section 18.7 Beats: Interference in Time

51. In certain ranges of a piano keyboard, more than one string is tuned to the same note to provide extra loudness. For example, the note at 110 Hz has two strings at this frequency. If one string slips from its normal tension of 600 N to 540 N, what beat frequency is heard when the hammer strikes the two strings simultaneously?

52. While attempting to tune the note C at 523 Hz, a piano tuner hears 2 beats/s between a reference oscillator and the string. (a) What are the possible frequencies of the string? (b) When she tightens the string slightly, she hears 3 beats/s. What is the frequency of the string now? (c) By what percentage should the piano tuner now change the tension in the string to bring it into tune?

53. A student holds a tuning fork oscillating at 256 Hz. He walks toward a wall at a constant speed of 1.33 m/s. (a) What beat frequency does he observe between the tuning fork and its echo? (b) How fast must he walk away from the wall to observe a beat frequency of 5.00 Hz?

54. When beats occur at a rate higher than about 20 per second, they are not heard individually but rather as a steady hum, called a combination tone. The player of a typical pipe organ can press a single key and make the organ produce sound with different fundamental frequencies. She can select and pull out different stops to make the same key for the note C produce sound at the following frequencies: 65.4 Hz from a so-called eight-foot pipe; \(2 \times 65.4 = 131\) Hz from a four-foot pipe; \(3 \times 65.4 = 196\) Hz from a two-and-two-thirds-foot pipe; \(4 \times 65.4 = 262\) Hz from a two-foot pipe; or any combination of these. With notes at low frequencies, she obtains sound with the richest quality by pulling out all the stops. When an air leak develops in one of the pipes, that pipe cannot be used. If a leak occurs in an eight-foot pipe, playing a combination of other pipes can create the sensation of sound at the frequency that the eight-foot pipe would produce. Which sets of stops, among those listed, could be pulled out to do this?

Section 18.8 Nonsinusoidal Wave Patterns

55. An A-major chord consists of the notes called A, C\(^4\), and E. It can be played on a piano by simultaneously striking strings with fundamental frequencies of 440.00 Hz, 554.37 Hz, and 659.26 Hz. The rich consonance of the chord is associated with near equality of the frequencies of some of the higher harmonics of the three tones. Consider the first five harmonics of each string and determine which harmonics show near equality.

56. Suppose that a flutist plays a 523-Hz C note with first harmonic displacement amplitude \(A_1 = 100\) nm. From Figure 18.24b read, by proportion, the displacement amplitudes of harmonics 2 through 7. Take these as the values \(A_2\) through \(A_7\) in the Fourier analysis of the sound, and assume that \(B_1 = B_2 = \cdots = B_7 = 0\). Construct a graph of the waveform of the sound. Your waveform will not look exactly like the flute waveform in Figure 18.23b because you simplify by ignoring cosine terms; nevertheless, it produces the same sensation to human hearing.

Additional Problems

57. On a marimba (Fig. P18.57), the wooden bar that sounds a tone when struck vibrates in a transverse standing wave having three antinodes and two nodes. The lowest frequency is note 87.0 Hz, produced by a bar 40.0 cm long. (a) Find the speed of transverse waves on the bar. (b) A resonant pipe suspended vertically below the center of the bar enhances the loudness of the emitted sound. If the pipe is open at the top end only and the speed of sound in air is 340 m/s, what is the length of the pipe required to resonate with the bar in part (a)?

![Figure P18.57 Marimba players in Mexico City.](image)

58. A loudspeaker at the front of a room and an identical loudspeaker at the rear of the room are being driven by the same oscillator at 456 Hz. A student walks at a uniform rate of 1.50 m/s along the length of the room. She hears a single tone, repeatedly becoming louder and softer. (a) Model these variations as beats between the Doppler-shifted sounds the student receives. Calculate the number of beats the student hears each second. (b) What If? Model the two speakers as producing a standing wave in the room and the student as walking between antinodes. Calculate the number of intensity maxima the student hears each second.

59. Two train whistles have identical frequencies of 180 Hz. When one train is at rest in the station and the other is...
moving nearby, a commuter standing on the station platform hears beats with a frequency of 2.00 beats/s when the whistles sound at the same time. What are the two possible speeds and directions that the moving train can have?

60. A string fixed at both ends and having a mass of 4.80 g, a length of 2.00 m, and a tension of 48.0 N vibrates in its second (n = 2) normal mode. What is the wavelength in air of the sound emitted by this vibrating string?

61. A student uses an audio oscillator of adjustable frequency to measure the depth of a water well. The student hears two successive resonances at 51.5 Hz and 60.0 Hz. How deep is the well?

62. A string has a mass per unit length of 9.00 \times 10^{-3} \text{ kg/m} and a length of 0.400 m. What must be the tension in the string if its second harmonic has the same frequency as the second resonance mode of a 1.75-m-long pipe open at one end?

63. Two wires are welded together end to end. The wires are made of the same material, but the diameter of one is twice that of the other. They are subjected to a tension of 4.60 N. The thin wire has a length of 40.0 cm and a linear mass density of 2.00 g/m. The combination is fixed at both ends and vibrated in such a way that two antinodes are present, with the node between them being right at the weld. (a) What is the frequency of vibration? (b) How long is the thick wire?

64. Review problem. For the arrangement shown in Figure P18.64, \( \theta = 30.0^\circ \), the inclined plane and the small pulley are frictionless, the string supports the object of mass \( M \) at the bottom of the plane, and the string has mass \( m \) that is small compared to \( M \). The system is in equilibrium and the vertical part of the string has a length \( h \). Standing waves are set up in the vertical section of the string. (a) Find the tension in the string. (b) Model the shape of the string as one leg and the hypotenuse of a right triangle. Find the whole length of the string. (c) Find the mass per unit length of the string. (d) Find the speed of waves on the string. (e) Find the lowest frequency for a standing wave. (f) Find the period of the standing wave having three nodes. (g) Find the wavelength of the standing wave having three nodes. (h) Find the frequency of the beats resulting from the interference of the sound wave of lowest frequency generated by the string with another sound wave having a frequency that is 2.00% greater.

65. A standing wave is set up in a string of variable length and tension by a vibrator of variable frequency. Both ends of the string are fixed. When the vibrator has a frequency \( f \), a string of length \( L \) and under tension \( T \), \( n \) antinodes are set up in the string. (a) If the length of the string is doubled, by what factor should the frequency be changed so that the same number of antinodes is produced? (b) If the frequency and length are held constant, what tension will produce \( n + 1 \) antinodes? (c) If the frequency is tripled and the length of the string is halved, by what factor should the tension be changed so that twice as many antinodes are produced?

66. A 0.010 0-kg wire, 2.00 m long, is fixed at both ends and vibrates in its simplest mode under a tension of 200 N. When a vibrating tuning fork is placed near the wire, a beat frequency of 5.00 Hz is heard. (a) What could be the frequency of the tuning fork? (b) What should the tension in the wire be if the beats are to disappear?

67. Two waves are described by the wave functions

\[ y_1(x, t) = 5.0 \sin(2.0x - 10t) \]

and

\[ y_2(x, t) = 10 \cos(2.0x - 10t) \]

where \( y_1, y_2, \) and \( x \) are in meters and \( t \) is in seconds. Show that the wave resulting from their superposition is also sinusoidal. Determine the amplitude and phase of this sinusoidal wave.

68. The wave function for a standing wave is given in Equation 18.3 as \( y = 2A \sin kx \cos \omega t \). (a) Rewrite this wave function in terms of the wavelength \( \lambda \) and the wave speed \( v \) of the wave. (b) Write the wave function of the simplest standing-wave vibration of a stretched string of length \( L \). (c) Write the wave function for the second harmonic. (d) Generalize these results and write the wave function for the \( n \)th resonance vibration.

69. Review problem. A 12.0-kg object hangs in equilibrium from a string with a total length of \( L = 5.00 \text{ m} \) and a linear mass density of \( \mu = 0.00100 \text{ kg/m} \). The string is wrapped around two light, frictionless pulleys that are separated by a distance of \( d = 2.00 \text{ m} \) (Fig. P18.69a). (a) Determine the tension in the string. (b) At what frequency must the string between the pulleys vibrate in order to form the standing wave pattern shown in Figure P18.69b?

![Figure P18.64](image_url)

![Figure P18.69](image_url)
70. A quartz watch contains a crystal oscillator in the form of a block of quartz that vibrates by contracting and expanding. Two opposite faces of the block, 7.05 mm apart, are antinodes, moving alternately toward each other and away from each other. The plane halfway between these two faces is a node of the vibration. The speed of sound in quartz is 3.70 km/s. Find the frequency of the vibration. An oscillating electric voltage accompanies the mechanical oscillation—the quartz is described as piezoelectric. An electric circuit feeds in energy to maintain the oscillation and also counts the voltage pulses to keep time.

Answers to Quick Quizzes

18.1 The shape of the string at \( t = 0.6 \) s is shown below.

18.2 (c). The pulses completely cancel each other in terms of displacement of elements of the string from equilibrium, but the string is still moving. A short time later, the string will be displaced again and the pulses will have passed each other.

18.3 (a). The pattern shown at the bottom of Figure 18.9a corresponds to the extreme position of the string. All elements of the string have momentarily come to rest.

18.4 (d). Near a nodal point, elements on one side of the point are moving upward at this instant and elements on the other side are moving downward.

18.5 (d). Choice (a) is incorrect because the number of nodes is one greater than the number of antinodes. Choice (b) is only true for half of the modes; it is not true for any odd-numbered mode. Choice (c) would be correct if we replace the word \textit{nodes} with \textit{antinodes}.

18.6 For each natural frequency of the glass, the standing wave must “fit” exactly around the rim. In Figure 18.17a we see three antinodes on the near side of the glass, and thus there must be another three on the far side. This corresponds to three complete waves. In a top view, the wave pattern looks like this (although we have greatly exaggerated the amplitude):

18.7 (b). With both ends open, the pipe has a fundamental frequency given by Equation 18.11: \( f_{\text{open}} = \frac{v}{2L} \). With one end closed, the pipe has a fundamental frequency given by Equation 18.12:

\[
f_{\text{closed}} = \frac{v}{4L} = \frac{v}{2L} = \frac{1}{2} f_{\text{open}}
\]

18.8 (c). The increase in temperature causes the speed of sound to go up. According to Equation 18.11, this will result in an increase in the fundamental frequency of a given organ pipe.

18.9 (b). Tightening the string has caused the frequencies to be farther apart, based on the increase in the beat frequency.
Thermodynamics

We now direct our attention to the study of thermodynamics, which involves situations in which the temperature or state (solid, liquid, gas) of a system changes due to energy transfers. As we shall see, thermodynamics is very successful in explaining the bulk properties of matter and the correlation between these properties and the mechanics of atoms and molecules.

Historically, the development of thermodynamics paralleled the development of the atomic theory of matter. By the 1820s, chemical experiments had provided solid evidence for the existence of atoms. At that time, scientists recognized that a connection between thermodynamics and the structure of matter must exist. In 1827, the botanist Robert Brown reported that grains of pollen suspended in a liquid move erratically from one place to another, as if under constant agitation. In 1905, Albert Einstein used kinetic theory to explain the cause of this erratic motion, which today is known as Brownian motion. Einstein explained this phenomenon by assuming that the grains are under constant bombardment by “invisible” molecules in the liquid, which themselves move erratically. This explanation gave scientists insight into the concept of molecular motion and gave credence to the idea that matter is made up of atoms. A connection was thus forged between the everyday world and the tiny, invisible building blocks that make up this world.

Thermodynamics also addresses more practical questions. Have you ever wondered how a refrigerator is able to cool its contents, what types of transformations occur in a power plant or in the engine of your automobile, or what happens to the kinetic energy of a moving object when the object comes to rest? The laws of thermodynamics can be used to provide explanations for these and other phenomena.
Why would someone designing a pipeline include these strange loops? Pipelines carrying liquids often contain loops such as these to allow for expansion and contraction as the temperature changes. We will study thermal expansion in this chapter. (Lowell Georgia/CORBIS)
In our study of mechanics, we carefully defined such concepts as mass, force, and kinetic energy to facilitate our quantitative approach. Likewise, a quantitative description of thermal phenomena requires careful definitions of such important terms as temperature, heat, and internal energy. This chapter begins with a discussion of temperature and with a description of one of the laws of thermodynamics (the so-called “zeroth law”).

Next, we consider why an important factor when we are dealing with thermal phenomena is the particular substance we are investigating. For example, gases expand appreciably when heated, whereas liquids and solids expand only slightly.

This chapter concludes with a study of ideal gases on the macroscopic scale. Here, we are concerned with the relationships among such quantities as pressure, volume, and temperature. In Chapter 21, we shall examine gases on a microscopic scale, using a model that represents the components of a gas as small particles.

19.1 Temperature and the Zeroth Law of Thermodynamics

We often associate the concept of temperature with how hot or cold an object feels when we touch it. Thus, our senses provide us with a qualitative indication of temperature. However, our senses are unreliable and often mislead us. For example, if we remove a metal ice tray and a cardboard box of frozen vegetables from the freezer, the ice tray feels colder than the box even though both are at the same temperature. The two objects feel different because metal transfers energy by heat at a higher rate than cardboard does. What we need is a reliable and reproducible method for measuring the relative hotness or coldness of objects rather than the rate of energy transfer. Scientists have developed a variety of thermometers for making such quantitative measurements.

We are all familiar with the fact that two objects at different initial temperatures eventually reach some intermediate temperature when placed in contact with each other. For example, when hot water and cold water are mixed in a bathtub, the final temperature of the mixture is somewhere between the initial hot and cold temperatures. Likewise, when an ice cube is dropped into a cup of hot coffee, it melts and the coffee’s temperature decreases.

To understand the concept of temperature, it is useful to define two often-used phrases: thermal contact and thermal equilibrium. To grasp the meaning of thermal contact, imagine that two objects are placed in an insulated container such that they interact with each other but not with the environment. If the objects are at different temperatures, energy is exchanged between them, even if they are initially not in physical contact with each other. The energy transfer mechanisms from Chapter 7 that we will focus on are heat and electromagnetic radiation. For purposes of the current discussion, we assume that two objects are in thermal contact with each other if energy can be exchanged between them by these processes due to a temperature difference.
Thermal equilibrium is a situation in which two objects would not exchange energy by heat or electromagnetic radiation if they were placed in thermal contact.

Let us consider two objects A and B, which are not in thermal contact, and a third object C, which is our thermometer. We wish to determine whether A and B are in thermal equilibrium with each other.

The thermometer (object C) is first placed in thermal contact with object A until thermal equilibrium is reached, as shown in Figure 19.1a. From that moment on, the thermometer’s reading remains constant, and we record this reading. The thermometer is then removed from object A and placed in thermal contact with object B, as shown in Figure 19.1b. The reading is again recorded after thermal equilibrium is reached. If the two readings are the same, then object A and object B are in thermal equilibrium with each other. If they are placed in contact with each other as in Figure 19.1c, there is no exchange of energy between them.

We can summarize these results in a statement known as the zeroth law of thermodynamics (the law of equilibrium):

Zeroth law of thermodynamics

If objects A and B are separately in thermal equilibrium with a third object C, then A and B are in thermal equilibrium with each other.

This statement can easily be proved experimentally and is very important because it enables us to define temperature. We can think of temperature as the property that determines whether an object is in thermal equilibrium with other objects. Two objects in thermal equilibrium with each other are at the same temperature. Conversely, if two objects have different temperatures, then they are not in thermal equilibrium with each other.

Quick Quiz 19.1 Two objects, with different sizes, masses, and temperatures, are placed in thermal contact. Energy travels (a) from the larger object to the smaller object (b) from the object with more mass to the one with less (c) from the object at higher temperature to the object at lower temperature.

1 We assume that negligible energy transfers between the thermometer and object A during the equilibrium process. Without this assumption, which is also made for the thermometer and object B, the measurement of the temperature of an object disturbs the system so that the measured temperature is different from the initial temperature of the object. In practice, whenever you measure a temperature with a thermometer, you measure the disturbed system, not the original system.
Thermometers and the Celsius Temperature Scale

Thermometers are devices that are used to measure the temperature of a system. All thermometers are based on the principle that some physical property of a system changes as the system’s temperature changes. Some physical properties that change with temperature are (1) the volume of a liquid, (2) the dimensions of a solid, (3) the pressure of a gas at constant volume, (4) the volume of a gas at constant pressure, (5) the electric resistance of a conductor, and (6) the color of an object. A temperature scale can be established on the basis of any one of these physical properties.

A common thermometer in everyday use consists of a mass of liquid—usually mercury or alcohol—that expands into a glass capillary tube when heated (Fig. 19.2). In this case the physical property that changes is the volume of a liquid. Any temperature change in the range of the thermometer can be defined as being proportional to the change in length of the liquid column. The thermometer can be calibrated by placing it in thermal contact with some natural systems that remain at constant temperature. One such system is a mixture of water and ice in thermal equilibrium at atmospheric pressure. On the Celsius temperature scale, this mixture is defined to have a temperature of zero degrees Celsius, which is written as 0°C; this temperature is called the ice point of water. Another commonly used system is a mixture of water and steam in thermal equilibrium at atmospheric pressure; its temperature is 100°C, which is the steam point of water. Once the liquid levels in the thermometer have been established at these two points, the length of the liquid column between the two points is divided into 100 equal segments to create the Celsius scale. Thus, each segment denotes a change in temperature of one Celsius degree.

Thermometers calibrated in this way present problems when extremely accurate readings are needed. For instance, the readings given by an alcohol thermometer calibrated at the ice and steam points of water might agree with those given by a mercury thermometer only at the calibration points. Because mercury and alcohol have different thermal expansion properties, when one thermometer reads a temperature of, for example, 50°C, the other may indicate a slightly different value. The discrepancies...
between thermometers are especially large when the temperatures to be measured are far from the calibration points.\(^2\)

An additional practical problem of any thermometer is the limited range of temperatures over which it can be used. A mercury thermometer, for example, cannot be used below the freezing point of mercury, which is \(-39^\circ\text{C}\), and an alcohol thermometer is not useful for measuring temperatures above \(85^\circ\text{C}\), the boiling point of alcohol. To surmount this problem, we need a universal thermometer whose readings are independent of the substance used in it. The gas thermometer, discussed in the next section, approaches this requirement.

### 19.3 The Constant-Volume Gas Thermometer and the Absolute Temperature Scale

One version of a gas thermometer is the constant-volume apparatus shown in Figure 19.3. The physical change exploited in this device is the variation of pressure of a fixed volume of gas with temperature. When the constant-volume gas thermometer was developed, it was calibrated by using the ice and steam points of water as follows. (A different calibration procedure, which we shall discuss shortly, is now used.) The flask was immersed in an ice-water bath, and mercury reservoir \(B\) was raised or lowered until the top of the mercury in column \(A\) was at the zero point on the scale. The height \(h\), the difference between the mercury levels in reservoir \(B\) and column \(A\), indicated the pressure in the flask at \(0^\circ\text{C}\).

The flask was then immersed in water at the steam point, and reservoir \(B\) was readjusted until the top of the mercury in column \(A\) was again at zero on the scale; this ensured that the gas’s volume was the same as it was when the flask was in the ice bath (hence, the designation “constant volume”). This adjustment of reservoir \(B\) gave a value for the gas pressure at \(100^\circ\text{C}\). These two pressure and temperature values were then plotted, as shown in Figure 19.4. The line connecting the two points serves as a calibration curve for unknown temperatures. (Other experiments show that a linear relationship between pressure and temperature is a very good assumption.) If we wanted to measure the temperature of a substance, we would place the gas flask in thermal contact with the substance and adjust the height of reservoir \(B\) until the top of the mercury column in \(A\) is at zero on the scale. The height of the mercury column indicates the pressure of the gas; knowing the pressure, we could find the temperature of the substance using the graph in Figure 19.4.

Now let us suppose that temperatures are measured with gas thermometers containing different gases at different initial pressures. Experiments show that the thermometer readings are nearly independent of the type of gas used, as long as the gas pressure is low and the temperature is well above the point at which the gas liquefies (Fig. 19.5). The agreement among thermometers using various gases improves as the pressure is reduced.

\(^2\) Two thermometers that use the same liquid may also give different readings. This is due in part to difficulties in constructing uniform-bore glass capillary tubes.
If we extend the straight lines in Figure 19.5 toward negative temperatures, we find a remarkable result—**in every case, the pressure is zero when the temperature is \(-273.15\)°C**! This suggests some special role that this particular temperature must play. It is used as the basis for the **absolute temperature scale**, which sets \(-273.15\)°C as its zero point. This temperature is often referred to as **absolute zero**. The size of a degree on the absolute temperature scale is chosen to be identical to the size of a degree on the Celsius scale. Thus, the conversion between these temperatures is

\[
T_C = T - 273.15
\]

where \(T_C\) is the Celsius temperature and \(T\) is the absolute temperature.

Because the ice and steam points are experimentally difficult to duplicate, an absolute temperature scale based on two new fixed points was adopted in 1954 by the International Committee on Weights and Measures. The first point is absolute zero. The second reference temperature for this new scale was chosen as the **triple point of water**, which is the single combination of temperature and pressure at which liquid water, gaseous water, and ice (solid water) coexist in equilibrium. This triple point occurs at a temperature of 0.01°C and a pressure of 4.58 mm of mercury. On the new scale, which uses the unit Kelvin, the temperature of water at the triple point was set at 273.16 kelvins, abbreviated 273.16 K. This choice was made so that the old absolute temperature scale based on the ice and steam points would agree closely with the new scale based on the triple point. This new absolute temperature scale (also called the **Kelvin scale**) employs the SI unit of absolute temperature, the Kelvin, which is defined to be \(1/273.16\) of the difference between absolute zero and the temperature of the triple point of water.

Figure 19.6 shows the absolute temperature for various physical processes and structures. The temperature of absolute zero (0 K) cannot be achieved, although laboratory experiments incorporating the laser cooling of atoms have come very close.

What would happen to a gas if its temperature could reach 0 K (and if it did not liquefy or solidify)? As Figure 19.5 indicates, the pressure it exerts on the walls of its container would be zero. In Chapter 21 we shall show that the pressure of a gas is proportional to the average kinetic energy of its molecules. Thus, according to classical physics, the kinetic energy of the gas molecules would become zero at absolute zero, and molecular motion would cease; hence, the molecules would settle out on the bottom of the container. Quantum theory modifies this prediction and shows that some residual energy, called the zero-point energy, would remain at this low temperature.

### The Celsius, Fahrenheit, and Kelvin Temperature Scales

Equation 19.1 shows that the Celsius temperature \(T_C\) is shifted from the absolute (Kelvin) temperature \(T\) by 273.15°. Because the size of a degree is the same on the two scales, a temperature difference of 5°C is equal to a temperature difference of 5 K. The two scales differ only in the choice of the zero point. Thus, the ice-point temperature on the Kelvin scale, 273.15 K, corresponds to 0.00°C, and the Kelvin-scale steam point, 373.15 K, is equivalent to 100.00°C.

A common temperature scale in everyday use in the United States is the **Fahrenheit scale**. This scale sets the temperature of the ice point at 32°F and the temperature of the steam point at 212°F. The relationship between the Celsius and Fahrenheit temperature scales is

\[
T_F = \frac{9}{5} T_C + 32^\circ F
\]

We can use Equations 19.1 and 19.2 to find a relationship between changes in temperature on the Celsius, Kelvin, and Fahrenheit scales:

\[
\Delta T_C = \Delta T = \frac{5}{9} \Delta T_F
\]

---

**PITFALL PREVENTION**

19.1 A Matter of Degree

Note that notations for temperatures in the Kelvin scale do not use the degree sign. The unit for a Kelvin temperature is simply “kelvins” and not “degrees Kelvin.”

---

3 Named after Anders Celsius (1701–1744), Daniel Gabriel Fahrenheit (1686–1736), and William Thomson, Lord Kelvin (1824–1907), respectively.
19.4 Thermal Expansion of Solids and Liquids

Our discussion of the liquid thermometer makes use of one of the best-known changes in a substance: as its temperature increases, its volume increases. This phenomenon, known as thermal expansion, has an important role in numerous engineering applications. For example, thermal-expansion joints, such as those shown in Figure 19.7, must be included in buildings, concrete highways, railroad tracks, brick walls, and bridges to compensate for dimensional changes that occur as the temperature changes.

Of the three temperature scales that we have discussed, only the Kelvin scale is based on a true zero value of temperature. The Celsius and Fahrenheit scales are based on an arbitrary zero associated with one particular substance—water—on one particular planet—Earth. Thus, if you encounter an equation that calls for a temperature $T$ or involves a ratio of temperatures, you must convert all temperatures to kelvins. If the equation contains a change in temperature $\Delta T$, using Celsius temperatures will give you the correct answer, in light of Equation 19.3, but it is always safest to convert temperatures to the Kelvin scale.

Quick Quiz 19.2 Consider the following pairs of materials. Which pair represents two materials, one of which is twice as hot as the other? (a) boiling water at 100°C, a glass of water at 50°C (b) boiling water at 100°C, frozen methane at $-50°C$ (c) an ice cube at $-20°C$, flames from a circus fire-eater at 233°C (d) No pair represents materials one of which is twice as hot as the other

Example 19.1 Converting Temperatures

On a day when the temperature reaches 50°F, what is the temperature in degrees Celsius and in kelvins?

**Solution** Substituting into Equation 19.2, we obtain

$$ T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(50 - 32) = 10°C $$

A convenient set of weather-related temperature equivalents to keep in mind is that 0°C is (literally) freezing at 32°F, 10°C is cool at 50°F, 20°C is room temperature, 30°C is warm at 86°F, and 40°C is a hot day at 104°F.

Example 19.2 Heating a Pan of Water

A pan of water is heated from 25°C to 80°C. What is the change in its temperature on the Kelvin scale and on the Fahrenheit scale?

**Solution** From Equation 19.3, we see that the change in temperature on the Celsius scale equals the change on the Kelvin scale. Therefore,

$$ \Delta T = \Delta T_C = 80°C - 25°C = 55°C = 55 K $$

From Equation 19.3, we also find that

$$ \Delta T_F = \frac{9}{5} \Delta T_C = \frac{9}{5}(55°C) = 99°F $$

19.4 Thermal Expansion of Solids and Liquids

Our discussion of the liquid thermometer makes use of one of the best-known changes in a substance: as its temperature increases, its volume increases. This phenomenon, known as thermal expansion, has an important role in numerous engineering applications. For example, thermal-expansion joints, such as those shown in Figure 19.7, must be included in buildings, concrete highways, railroad tracks, brick walls, and bridges to compensate for dimensional changes that occur as the temperature changes.

Thermal expansion is a consequence of the change in the average separation between the atoms in an object. To understand this, model the atoms as being connected by stiff springs, as discussed in Section 15.3 and shown in Figure 15.12b. At ordinary temperatures, the atoms in a solid oscillate about their equilibrium positions with an amplitude of approximately $10^{-11}$ m and a frequency of approximately $10^{15}$ Hz. The average spacing between the atoms is about $10^{-10}$ m. As the temperature of the solid
increases, the atoms oscillate with greater amplitudes; as a result, the average separation between them increases. Consequently, the object expands.

If thermal expansion is sufficiently small relative to an object’s initial dimensions, the change in any dimension is, to a good approximation, proportional to the first power of the temperature change. Suppose that an object has an initial length $L_i$ along some direction at some temperature and that the length increases by an amount $\Delta L$ for a change in temperature $\Delta T$. Because it is convenient to consider the fractional change in length per degree of temperature change, we define the average coefficient of linear expansion as

$$\alpha = \frac{\Delta L}{L_i \Delta T}$$

Experiments show that $\alpha$ is constant for small changes in temperature. For purposes of calculation, this equation is usually rewritten as

$$\Delta L = \alpha L_i \Delta T$$  \hfill (19.4)$$

or as

$$L_f - L_i = \alpha L_i (T_f - T_i)$$  \hfill (19.5)$$

where $L_f$ is the final length, $T_i$ and $T_f$ are the initial and final temperatures, and the proportionality constant $\alpha$ is the average coefficient of linear expansion for a given material and has units of $(^\circ\text{C})^{-1}$.

It may be helpful to think of thermal expansion as an effective magnification or as a photographic enlargement of an object. For example, as a metal washer is heated (Fig. 19.8), all dimensions, including the radius of the hole, increase according to Equation 19.4. Notice that this is equivalent to saying that a cavity in a piece of material expands in the same way as if the cavity were filled with the material.

Table 19.1 lists the average coefficient of linear expansion for various materials. Note that for these materials $\alpha$ is positive, indicating an increase in length with increasing temperature. This is not always the case. Some substances—calcite ($\text{CaCO}_3$) is one example—expand along one dimension (positive $\alpha$) and contract along another (negative $\alpha$) as their temperatures are increased.

4 More precisely, thermal expansion arises from the asymmetrical nature of the potential-energy curve for the atoms in a solid, as shown in Figure 15.12a. If the oscillators were truly harmonic, the average atomic separations would not change regardless of the amplitude of vibration.
Because the linear dimensions of an object change with temperature, it follows that surface area and volume change as well. The change in volume is proportional to the initial volume \( V_i \) and to the change in temperature according to the relationship

\[
\Delta V = \beta V_i \Delta T
\]  

(19.6)

where \( \beta \) is the average coefficient of volume expansion. For a solid, the average coefficient of volume expansion is three times the average linear expansion coefficient: \( \beta = 3\alpha \). (This assumes that the average coefficient of linear expansion of the solid is the same in all directions—that is, the material is isotropic.)

To see that \( \beta = 3\alpha \) for a solid, consider a solid box of dimensions \( \ell, w, \) and \( h \). Its volume at some temperature \( T_i \) is \( V_i = \ell w h \). If the temperature changes to \( T_i + \Delta T \), its volume changes to \( V_i + \Delta V \), where each dimension changes according to Equation 19.4. Therefore,

\[
V_i + \Delta V = (\ell + \Delta\ell)(w + \Delta w)(h + \Delta h)
\]

\[
= (\ell + \alpha\ell \Delta T)(w + \alpha w \Delta T)(h + \alpha h \Delta T)
\]

\[
= \ell w h (1 + \alpha \Delta T)^3
\]

If we now divide both sides by \( V_i \) and isolate the term \( \Delta V/V_i \), we obtain the fractional change in volume:

\[
\frac{\Delta V}{V_i} = 3\alpha \Delta T + 3(\alpha \Delta T)^2 + (\alpha \Delta T)^3
\]

Because \( \alpha\Delta T \ll 1 \) for typical values of \( \Delta T \) (\( \sim 100^\circ C \)), we can neglect the terms \( 3(\alpha \Delta T)^2 \) and \( (\alpha \Delta T)^3 \). Upon making this approximation, we see that

\[
\frac{\Delta V}{V_i} = 3\alpha \Delta T
\]

\[
3\alpha = \frac{\Delta V}{V_i} \frac{1}{\Delta T}
\]

Equation 19.6 shows that the right side of this expression is equal to \( \beta \), and so we have \( 3\alpha = \beta \), the relationship we set out to prove. In a similar way, you can show that the change in area of a rectangular plate is given by \( \Delta A = 2\alpha A_i \Delta T \) (see Problem 55).

As Table 19.1 indicates, each substance has its own characteristic average coefficient of expansion. For example, when the temperatures of a brass rod and a steel rod of
equal length are raised by the same amount from some common initial value, the brass rod expands more than the steel rod does because brass has a greater average coefficient of expansion than steel does. A simple mechanism called a bimetallic strip utilizes this principle and is found in practical devices such as thermostats. It consists of two thin strips of dissimilar metals bonded together. As the temperature of the strip increases, the two metals expand by different amounts and the strip bends, as shown in Figure 19.9.

![Figure 19.9](a) A bimetallic strip bends as the temperature changes because the two metals have different expansion coefficients. (b) A bimetallic strip used in a thermostat to break or make electrical contact.

Quick Quiz 19.3 If you are asked to make a very sensitive glass thermometer, which of the following working liquids would you choose? (a) mercury (b) alcohol (c) gasoline (d) glycerin

Quick Quiz 19.4 Two spheres are made of the same metal and have the same radius, but one is hollow and the other is solid. The spheres are taken through the same temperature increase. Which sphere expands more? (a) solid sphere (b) hollow sphere (c) They expand by the same amount. (d) not enough information to say

Example 19.3 Expansion of a Railroad Track

A segment of steel railroad track has a length of 30.000 m when the temperature is 0.0°C.

(A) What is its length when the temperature is 40.0°C?

Solution Making use of Table 19.1 and noting that the change in temperature is 40.0°C, we find that the increase in length is

\[ \Delta L = aL_i \Delta T = (11 \times 10^{-6} \text{C}^{-1})(30.000 \text{ m})(40.0 \text{ C}) = 0.013 \text{ m} \]

If the track is 30.000 m long at 0.0°C, its length at 40.0°C is 30.013 m.

(B) Suppose that the ends of the rail are rigidly clamped at 0.0°C so that expansion is prevented. What is the thermal stress set up in the rail if its temperature is raised to 40.0°C?

Solution The thermal stress will be the same as that in the situation in which we allow the rail to expand freely and then compress it with a mechanical force F back to its original length. From the definition of Young’s modulus for a solid (see Eq. 12.6), we have

\[ \text{Tensile stress} = \frac{F}{A} = Y \frac{\Delta L}{L_i} \]

Because Y for steel is $20 \times 10^{10}$ N/m² (see Table 12.1), we have
The Unusual Behavior of Water

Liquids generally increase in volume with increasing temperature and have average coefficients of volume expansion about ten times greater than those of solids. Cold water is an exception to this rule, as we can see from its density-versus-temperature curve, shown in Figure 19.11. As the temperature increases from 0°C to 4°C, water contracts and thus its density increases. Above 4°C, water expands with increasing temperature, and so its density decreases. Thus, the density of water reaches a maximum value of 1.000 g/cm³ at 4°C.

We can use this unusual thermal-expansion behavior of water to explain why a pond begins freezing at the surface rather than at the bottom. When the atmospheric temperature drops from, for example, 7°C to 6°C, the surface water also cools and consequently decreases in volume. This means that the surface water is denser than the water below it, which has not cooled and decreased in volume. As a result, the surface water sinks, and warmer water from below is forced to the surface to be cooled. When the atmospheric temperature is between 4°C and 0°C, however, the surface water expands as it cools, becoming less dense than the water below it. The mixing process...
stops, and eventually the surface water freezes. As the water freezes, the ice remains on the surface because ice is less dense than water. The ice continues to build up at the surface, while water near the bottom remains at 4°C. If this were not the case, then fish and other forms of marine life would not survive.

19.5 Macroscopic Description of an Ideal Gas

The volume expansion equation \( \Delta V = \beta V_i \Delta T \) is based on the assumption that the material has an initial volume \( V_i \) before the temperature change occurs. This is the case for solids and liquids because they have a fixed volume at a given temperature.

The case for gases is completely different. The interatomic forces within gases are very weak, and, in many cases, we can imagine these forces to be nonexistent and still make very good approximations. Note that there is no equilibrium separation for the atoms and, thus, no “standard” volume at a given temperature. As a result, we cannot express changes in volume \( \Delta V \) in a process on a gas with Equation 19.6 because we have no defined volume \( V_i \) at the beginning of the process. For a gas, the volume is entirely determined by the container holding the gas. Thus, equations involving gases will contain the volume \( V \) as a variable, rather than focusing on a change in the volume from an initial value.

For a gas, it is useful to know how the quantities volume \( V \), pressure \( P \), and temperature \( T \) are related for a sample of gas of mass \( m \). In general, the equation that interrelates these quantities, called the equation of state, is very complicated. However, if the gas is maintained at a very low pressure (or low density), the equation of state is quite simple and can be found experimentally. Such a low-density gas is commonly referred to as an ideal gas.\(^5\)

It is convenient to express the amount of gas in a given volume in terms of the number of moles \( n \). One mole of any substance is that amount of the substance that

\(^5\) To be more specific, the assumption here is that the temperature of the gas must not be too low (the gas must not condense into a liquid) or too high, and that the pressure must be low. The concept of an ideal gas implies that the gas molecules do not interact except upon collision, and that the molecular volume is negligible compared with the volume of the container. In reality, an ideal gas does not exist. However, the concept of an ideal gas is very useful because real gases at low pressures behave as ideal gases do.
contains **Avogadro's number** \(N_A = 6.022 \times 10^{23}\) of constituent particles (atoms or molecules). The number of moles \(n\) of a substance is related to its mass \(m\) through the expression

\[
n = \frac{m}{M}
\]

(19.7)

where \(M\) is the molar mass of the substance. The molar mass of each chemical element is the atomic mass (from the periodic table, Appendix C) expressed in g/mol. For example, the mass of one He atom is 4.00 \(u\) (atomic mass units), so the molar mass of He is 4.00 g/mol. For a molecular substance or a chemical compound, you can add up the molar mass from its molecular formula. The molar mass of stable diatomic oxygen \((O_2)\) is 32.0 g/mol.

Now suppose that an ideal gas is confined to a cylindrical container whose volume can be varied by means of a movable piston, as in Figure 19.12. If we assume that the cylinder does not leak, the mass (or the number of moles) of the gas remains constant. For such a system, experiments provide the following information. First, when the gas is kept at a constant temperature, its pressure is inversely proportional to its volume (Boyle’s law). Second, when the pressure of the gas is kept constant, its volume is directly proportional to its temperature (the law of Charles and Gay-Lussac). These observations are summarized by the **equation of state for an ideal gas**:

\[
PV = nRT
\]

(19.8)

In this expression, known as the **ideal gas law**, \(R\) is a constant and \(n\) is the number of moles of gas in the sample. Experiments on numerous gases show that as the pressure approaches zero, the quantity \(PV/nT\) approaches the same value \(R\) for all gases. For this reason, \(R\) is called the **universal gas constant**. In SI units, in which pressure is expressed in pascals (1 Pa = 1 N/m\(^2\)) and volume in cubic meters, the product \(PV\) has units of newton·meters, or joules, and \(R\) has the value

\[
R = 8.314 \text{ J/mol·K}
\]

(19.9)

If the pressure is expressed in atmospheres and the volume in liters (1 L = 10\(^3\) cm\(^3\) = 10\(^{-3}\) m\(^3\)), then \(R\) has the value

\[
R = 0.08214 \text{ L·atm/mol·K}
\]

Using this value of \(R\) and Equation 19.8, we find that the volume occupied by 1 mol of any gas at atmospheric pressure and at 0°C (273 K) is 22.4 L.

The ideal gas law states that if the volume and temperature of a fixed amount of gas do not change, then the pressure also remains constant. Consider a bottle of champagne that is shaken and then spews liquid when opened, as shown in Figure 19.13.
A common misconception is that the pressure inside the bottle is increased when the bottle is shaken. On the contrary, because the temperature of the bottle and its contents remains constant as long as the bottle is sealed, so does the pressure, as can be shown by replacing the cork with a pressure gauge. The correct explanation is as follows. Carbon dioxide gas resides in the volume between the liquid surface and the cork. Shaking the bottle displaces some of this carbon dioxide gas into the liquid, where it forms bubbles, and these bubbles become attached to the inside of the bottle. (No new gas is generated by shaking.) When the bottle is opened, the pressure is reduced; this causes the volume of the bubbles to increase suddenly. If the bubbles are attached to the bottle (beneath the liquid surface), their rapid expansion expels liquid from the bottle. If the sides and bottom of the bottle are first tapped until no bubbles remain beneath the surface, then when the champagne is opened, the drop in pressure will not force liquid from the bottle.

The ideal gas law is often expressed in terms of the total number of molecules \(N\). Because the total number of molecules equals the product of the number of moles \(n\) and Avogadro’s number \(N_A\), we can write Equation 19.8 as

\[
PV = nRT = \frac{N}{N_A} RT
\]

\[
PV = Nk_BT
\]  

(19.10)

where \(k_B\) is Boltzmann’s constant, which has the value

\[
k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K}
\]  

(19.11)

It is common to call quantities such as \(P\), \(V\), and \(T\) the thermodynamic variables of an ideal gas. If the equation of state is known, then one of the variables can always be expressed as some function of the other two.

**Quick Quiz 19.5** A common material for cushioning objects in packages is made by trapping bubbles of air between sheets of plastic. This material is more effective at keeping the contents of the package from moving around inside the package on (a) a hot day  (b) a cold day  (c) either hot or cold days.

**Quick Quiz 19.6** A helium-filled rubber balloon is left in a car on a cold winter night. Compared to its size when it was in the warm car the afternoon before, the size the next morning is (a) larger  (b) smaller  (c) unchanged.

**Quick Quiz 19.7** On a winter day, you turn on your furnace and the temperature of the air inside your home increases. Assuming that your home has the normal amount of leakage between inside air and outside air, the number of moles of air in your room at the higher temperature is (a) larger than before  (b) smaller than before  (c) the same as before.

### Example 19.5 How Many Moles of Gas in a Container?

An ideal gas occupies a volume of 100 cm\(^3\) at 20°C and 100 Pa. Find the number of moles of gas in the container.

**Solution** The quantities given are volume, pressure, and temperature: \(V = 100 \text{ cm}^3 = 1.00 \times 10^{-4} \text{ m}^3\), \(P = 100 \text{ Pa}\), and \(T = 20^\circ \text{C} = 293 \text{ K}\). Using Equation 19.8, we find that

\[
n = \frac{PV}{RT} = \frac{(100 \text{ Pa})(1.00 \times 10^{-4} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})}
\]

\[
= 4.11 \times 10^{-6} \text{ mol}
\]
Example 19.6 Filling a Scuba Tank

A certain scuba tank is designed to hold 66.0 ft³ of air when it is at atmospheric pressure at 22°C. When this volume of air is compressed to an absolute pressure of 3000 lb/in.² and stored in a 10.0-L (0.350-ft³) tank, the air becomes so hot that the tank must be allowed to cool before it can be used. Before the air cools, what is its temperature? (Assume that the air behaves like an ideal gas.)

Solution If no air escapes during the compression, then the number of moles \( n \) of air remains constant; therefore, using \( PV = nRT \), with \( n \) and \( R \) constant, we obtain a relationship between the initial and final values:

\[
\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}
\]

The initial pressure of the air is 14.7 lb/in.², its final pressure is 3000 lb/in.², and the air is compressed from an initial volume of 66.0 ft³ to a final volume of 0.350 ft³. The initial temperature, converted to SI units, is 295 K. Solving for \( T_f \), we obtain

\[
T_f = \left( \frac{P_f V_f}{P_i V_i} \right) T_i = \left( \frac{3000 \text{ lb/in.}^2}{14.7 \text{ lb/in.}^2} \right) \left( \frac{0.350 \text{ ft}^3}{66.0 \text{ ft}^3} \right) (295 \text{ K})
\]

\[
= 319 \text{ K}
\]

Example 19.7 Heating a Spray Can

A spray can containing a propellant gas at twice atmospheric pressure (202 kPa) and having a volume of 125.00 cm³ is at 22°C. It is then tossed into an open fire. When the temperature of the gas in the can reaches 195°C, what is the pressure inside the can? Assume any change in the volume of the can is negligible.

Solution We employ the same approach we used in Example 19.6, starting with the expression

\[
\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}
\]

Because the initial and final volumes of the gas are assumed to be equal, this expression reduces to

\[
\frac{P_i}{T_i} = \frac{P_f}{T_f}
\]

Solving for \( P_f \) gives

\[
P_f = \left( \frac{T_f}{T_i} \right) P_i = \left( \frac{468 \text{ K}}{295 \text{ K}} \right) (202 \text{ kPa}) = 320 \text{ kPa}
\]

Obviously, the higher the temperature, the higher the pressure exerted by the trapped gas. Of course, if the pressure increases sufficiently, the can will explode. Because of this possibility, you should never dispose of spray cans in a fire.

What If? Suppose we include a volume change due to thermal expansion of the steel can as the temperature increases. Does this alter our answer for the final pressure significantly?

Because the thermal expansion coefficient of steel is very small, we do not expect much of an effect on our final answer. The change in the volume of the can is found using Equation 19.6 and the value for \( \alpha \) for steel from Table 19.1:

\[
\Delta V = \beta V_i \Delta T = 3 \alpha V_i \Delta T
\]

\[
= 3(11 \times 10^{-6} \text{ °C}^{-1})(125.00 \text{ cm}^3)(173\text{°C})
\]

\[
= 0.71 \text{ cm}^3
\]

So the final volume of the can is 125.71 cm³. Starting from Equation (1) again, the equation for the final pressure becomes

\[
P_f = \left( \frac{T_f}{T_i} \right) \left( \frac{V_i}{V_f} \right) P_i
\]

This differs from Equation (2) only in the factor \( V_i/V_f \). Let us evaluate this factor:

\[
\frac{V_i}{V_f} = \frac{125.00 \text{ cm}^3}{125.71 \text{ cm}^3} = 0.994 = 99.4\%
\]

Thus, the final pressure will differ by only 0.6% from the value we calculated without considering the thermal expansion of the can. Taking 99.4% of the previous final pressure, the final pressure including thermal expansion is 318 kPa.

Explore this situation at the Interactive Worked Example link at http://www.pse6.com.

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

**SUMMARY**

Two objects are in thermal equilibrium with each other if they do not exchange energy when in thermal contact.

The zeroth law of thermodynamics states that if objects A and B are separately in thermal equilibrium with a third object C, then objects A and B are in thermal equilibrium with each other.

Temperature is the property that determines whether an object is in thermal equilibrium with other objects. Two objects in thermal equilibrium with each other are at the same temperature.
The SI unit of absolute temperature is the **kelvin**, which is defined to be the fraction 1/273.16 of the temperature of the triple point of water.

When the temperature of an object is changed by an amount \( \Delta T \), its length changes by an amount \( \Delta L \) that is proportional to \( \Delta T \) and to its initial length \( L_i \):

\[
\Delta L = \alpha L_i \Delta T
\]  

(19.4)

where the constant \( \alpha \) is the **average coefficient of linear expansion**. The **average coefficient of volume expansion** \( \beta \) for a solid is approximately equal to \( 3\alpha \).

An **ideal gas** is one for which \( PV/nT \) is constant. An ideal gas is described by the **equation of state**, \( PV = nRT \)  

(19.8)

where \( n \) equals the number of moles of the gas, \( V \) is its volume, \( R \) is the universal gas constant (8.314 J/mol·K), and \( T \) is the absolute temperature. A real gas behaves approximately as an ideal gas if it has a low density.

### Questions

1. Is it possible for two objects to be in thermal equilibrium if they are not in contact with each other? Explain.

2. A piece of copper is dropped into a beaker of water. If the water’s temperature rises, what happens to the temperature of the copper? Under what conditions are the water and copper in thermal equilibrium?

3. In describing his upcoming trip to the Moon, and as portrayed in the movie *Apollo 13* (Universal, 1995), astronaut Jim Lovell said, “I’ll be walking in a place where there’s a 400-degree difference between sunlight and shadow.” What is it that is hot in sunlight and cold in shadow? Suppose an astronaut standing on the Moon holds a thermometer in his gloved hand. Is it reading the temperature of the vacuum at the Moon’s surface? Does it read any temperature? If so, what object or substance has that temperature?

4. Rubber has a negative average coefficient of linear expansion. What happens to the size of a piece of rubber as it is warmed?

5. Explain why a column of mercury in a thermometer first descends slightly and then rises when the thermometer is placed into hot water.

6. Why should the amalgam used in dental fillings have the same average coefficient of expansion as a tooth? What would occur if they were mismatched?

7. Markings to indicate length are placed on a steel tape in a room that has a temperature of 22°C. Are measurements made with the tape on a day when the temperature is 27°C too long, too short, or accurate? Defend your answer.

8. Determine the number of grams in a mole of the following gases: (a) hydrogen (b) helium (c) carbon monoxide.

9. What does the ideal gas law predict about the volume of a sample of gas at absolute zero? Why is this prediction incorrect?

10. An inflated rubber balloon filled with air is immersed in a flask of liquid nitrogen that is at 77 K. Describe what happens to the balloon, assuming that it remains flexible while being cooled.

11. Two identical cylinders at the same temperature each contain the same kind of gas and the same number of moles of gas. If the volume of cylinder A is three times greater than the volume of cylinder B, what can you say about the relative pressures in the cylinders?

12. After food is cooked in a pressure cooker, why is it very important to cool off the container with cold water before attempting to remove the lid?

13. The shore of the ocean is very rocky at a particular place. The rocks form a cave sloping upward from an underwater opening, as shown in Figure Q19.13a. (a) Inside the cave is
a pocket of trapped air. As the level of the ocean rises and falls with the tides, will the level of water in the cave rise and fall? If so, will it have the same amplitude as that of the ocean? (b) What if? Now suppose that the cave is deeper in the water, so that it is completely submerged and filled with water at high tide, as shown in Figure Q19.13b. At low tide, will the level of water in the cave be the same as that of the ocean?

14. In Colonization: Second Contact (Harry Turtledove, Ballantine Publishing Group, 1999), the Earth has been partially settled by aliens from another planet, whom humans call Lizards. Laboratory study by humans of Lizard science requires “shifting back and forth between the metric system and the one the Lizards used, which was also based on powers of ten but used different basic quantities for everything but temperature.” Why might temperature be an exception?

15. The pendulum of a certain pendulum clock is made of brass. When the temperature increases, does the period of the clock increase, decrease, or remain the same? Explain.

16. An automobile radiator is filled to the brim with water while the engine is cool. What happens to the water when the engine is running and the water is heated? What do modern automobiles have in their cooling systems to prevent the loss of coolants?

17. Metal lids on glass jars can often be loosened by running hot water over them. How is this possible?

18. When the metal ring and metal sphere in Figure Q19.18 are both at room temperature, the sphere can just be passed through the ring. After the sphere is heated, it cannot be passed through the ring. Explain. What if? What if the ring is heated and the sphere is left at room temperature? Does the sphere pass through the ring?

---

**PROBLEMS**

1. 2, 3 = straightforward, intermediate, challenging  □ = full solution available in the Student Solutions Manual and Study Guide
   = coached solution with hints available at http://www.pse6.com  □ = computer useful in solving problem
   = paired numerical and symbolic problems

---

**Section 19.2 Thermometers and the Celsius Temperature Scale**

**Section 19.3 The Constant-Volume Gas Thermometer and the Absolute Temperature Scale**

1. A constant-volume gas thermometer is calibrated in dry ice (that is, carbon dioxide in the solid state, which has a temperature of −80.0°C) and in boiling ethyl alcohol (78.0°C). The two pressures are 0.900 atm and 1.635 atm.
   (a) What is the pressure at the freezing point of water and (c) the boiling point of water?

2. In a constant-volume gas thermometer, the pressure at 20.0°C is 0.980 atm. (a) What is the pressure at 45.0°C? (b) What is the temperature if the pressure is 0.500 atm?

3. Liquid nitrogen has a boiling point of −195.81°C at atmospheric pressure. Express this temperature (a) in degrees Fahrenheit and (b) in kelvins.

4. Convert the following to equivalent temperatures on the Celsius and Kelvin scales: (a) the normal human body temperature, 98.6°F; (b) the air temperature on a cold day, −5.00°F.

5. The temperature difference between the inside and the outside of an automobile engine is 450°C. Express this temperature difference on (a) the Fahrenheit scale and (b) the Kelvin scale.

6. On a Strange temperature scale, the freezing point of water is −15.0°S and the boiling point is +60.0°S. Develop a linear conversion equation between this temperature scale and the Celsius scale.

7. The melting point of gold is 1337°C and the boiling point is 3545°C. Develop the Fahrenheit and Celsius scales. (a) What is the Fahrenheit scale for 1337°C and the Celsius scale for 3545°C? (b) Compute the difference between these temperatures in Fahrenheit and in kelvins.

---

**Section 19.4 Thermal Expansion of Solids and Liquids**

**Note:** Table 19.1 is available for use in solving problems in this section.

8. The New River Gorge bridge in West Virginia is a steel arch bridge 518 m in length. How much does the total length of the roadway decking change between temperature extremes of −20.0°C and 35.0°C? The result indicates the
size of the expansion joints that must be built into the structure.

9. A copper telephone wire has essentially no sag between poles 35.0 m apart on a winter day when the temperature is $-20.0^\circ C$. How much longer is the wire on a summer day when $T_C = 35.0^\circ C$?

10. The concrete sections of a certain superhighway are designed to have a length of 25.0 m. The sections are poured and cured at 10.0°C. What minimum spacing should the engineer leave between the sections to eliminate buckling if the concrete is to reach a temperature of 50.0°C?

11. A pair of eyeglass frames is made of epoxy plastic. At room temperature (20.0°C), the frames have circular lens holes 2.20 cm in radius. To what temperature must the frames be heated if lenses 2.21 cm in radius are to be inserted in them? The average coefficient of linear expansion for epoxy is $1.30 \times 10^{-4} \, (^\circ C)^{-1}$.

12. Each year thousands of children are badly burned by hot tap water. Figure P19.12 shows a cross-sectional view of an antiscalding faucet attachment designed to prevent such accidents. Within the device, a spring made of material with a high coefficient of thermal expansion controls a movable plunger. When the water temperature rises above a preset safe value, the expansion of the spring causes the plunger to shut off the water flow. If the initial length $L$ of the unstressed spring is 2.40 cm and its coefficient of linear expansion is $22.0 \times 10^{-6} \, (^\circ C)^{-1}$, determine the increase in length of the spring when the water temperature rises by 30.0°C. (You will find the increase in length to be small. For this reason actual devices have a more complicated mechanical design, to provide a greater variation in valve opening for the temperature change anticipated.)

![Figure P19.12](image1)

13. The active element of a certain laser is made of a glass rod 30.0 cm long by 1.50 cm in diameter. If the temperature of the rod increases by 65.0°C, what is the increase in (a) its length, (b) its diameter, and (c) its volume? Assume that the average coefficient of linear expansion of the glass is $9.00 \times 10^{-6} \, (^\circ C)^{-1}$.

14. Review problem. Inside the wall of a house, an L-shaped section of hot-water pipe consists of a straight horizontal piece 28.0 cm long, an elbow, and a straight vertical piece 134 cm long (Figure P19.14). A stud and a second-story floorboard hold stationary the ends of this section of copper pipe. Find the magnitude and direction of the displacement of the pipe elbow when the water flow is turned on, raising the temperature of the pipe from 18.0°C to 46.5°C.

![Figure P19.14](image2)

15. A brass ring of diameter 10.00 cm at 20.0°C is heated and slipped over an aluminum rod of diameter 10.01 cm at 20.0°C. Assuming the average coefficients of linear expansion are constant, (a) to what temperature must this combination be cooled to separate them? Is this attainable? (b) What If? What if the aluminum rod were 10.02 cm in diameter?

16. A square hole 8.00 cm along each side is cut in a sheet of copper. (a) Calculate the change in the area of this hole if the temperature of the sheet is increased by 50.0 K. (b) Does this change represent an increase or a decrease in the area enclosed by the hole?

17. The average coefficient of volume expansion for carbon tetrachloride is $5.81 \times 10^{-4} \, (^\circ C)^{-1}$. If a 50.0-gal steel container is filled completely with carbon tetrachloride when the temperature is 10.0°C, how much will spill over when the temperature rises to 30.0°C?

18. At 20.0°C, an aluminum ring has an inner diameter of 5.000 0 cm and a brass rod has a diameter of 5.050 0 cm. (a) If only the ring is heated, what temperature must it reach so that it will just slip over the rod? (b) What If? If both are heated together, what temperature must they both reach so that the ring just slips over the rod? Would this latter process work?

19. A volumetric flask made of Pyrex is calibrated at 20.0°C. It is filled to the 100-mL mark with 35.0°C acetone. (a) What is the volume of the acetone when it cools to 20.0°C? (b) How significant is the change in volume of the flask?
20. A concrete walk is poured on a day when the temperature is 20.0°C in such a way that the ends are unable to move. (a) What is the stress in the cement on a hot day of 50.0°C? (b) Does the concrete fracture? Take Young’s modulus for concrete to be 7.00 × 10^9 N/m^2 and the compressive strength to be 2.00 × 10^9 N/m^2.

21. A hollow aluminum cylinder 20.0 cm deep has an internal capacity of 2.000 L at 20.0°C. It is completely filled with turpentine and then slowly warmed to 80.0°C. (a) How much turpentine overflows? (b) If the cylinder is then cooled back to 20.0°C, how far below the cylinder’s rim does the turpentine’s surface recede?

22. A beaker made of ordinary glass contains a lead sphere of diameter 4.00 cm firmly attached to its bottom. At a uniform temperature of −10.0°C, the beaker is filled to the brim with 118 cm^3 of mercury, which completely covers the sphere. How much mercury overflows from the beaker if the temperature is raised to 30.0°C?

23. A steel rod undergoes a stretching force of 500 N. Its cross-sectional area is 2.00 cm^2. Find the change in temperature that would elongate the rod by the same amount as the 500-N force does. Tables 12.1 and 19.1 are available to you.

24. The Golden Gate Bridge in San Francisco has a main span of length 1.28 km—one of the longest in the world. Imagine that a taut steel wire with this length and a cross-sectional area of 4.00 × 10^{-8} m^2 is laid on the bridge deck with its ends attached to the towers of the bridge, on a summer day when the temperature of the wire is 35.0°C. (a) When winter arrives, the towers stay the same distance apart and the bridge deck keeps the same shape as its expansion joints open. When the temperature drops to −10.0°C, what is the tension in the wire? Take Young’s modulus for steel to be 20.0 × 10^10 N/m^2. (b) Permanent deformation occurs if the stress in the steel exceeds its elastic limit of 3.00 × 10^8 N/m^2. At what temperature would this happen? (c) What If? How would your answers to (a) and (b) change if the Golden Gate Bridge were twice as long?

25. A certain telescope forms an image of part of a cluster of stars on a square silicon charge-coupled detector (CCD) chip 2.00 cm on each side. A star field is focused on the CCD chip when it is first turned on and its temperature is 20.0°C. The star field contains 5 342 stars scattered uniformly. To make the detector more sensitive, it is cooled to −100°C. How many star images then fit onto the chip? The average coefficient of linear expansion of silicon is 4.68 × 10^{-6} (°C)^{-1}.

Section 19.5 Macroscopic Description of an Ideal Gas

Note: Problem 8 in Chapter 1 can be assigned with this section.

26. Gas is contained in an 8.00-L vessel at a temperature of 20.0°C and a pressure of 9.00 atm. (a) Determine the number of moles of gas in the vessel. (b) How many molecules are there in the vessel?

27. An automobile tire is inflated with air originally at 10.0°C and normal atmospheric pressure. During the process, the air is compressed to 28.0% of its original volume and the temperature is increased to 40.0°C. (a) What is the tire pressure? (b) After the car is driven at high speed, the tire air temperature rises to 85.0°C and the interior volume of the tire increases by 2.00%. What is the new tire pressure (absolute) in pascals?

28. A tank having a volume of 0.100 m^3 contains helium gas at 150 atm. How many balloons can the tank blow up if each filled balloon is a sphere 0.300 m in diameter at an absolute pressure of 1.20 atm?

29. An auditorium has dimensions 10.0 m × 20.0 m × 30.0 m. How many molecules of air fill the auditorium at 20.0°C and a pressure of 101 kPa?

30. Imagine a baby alien playing with a spherical balloon the size of the Earth in the outer solar system. Helium gas inside the balloon has a uniform temperature of 50.0 K due to radiation from the Sun. The uniform pressure of the helium is equal to normal atmospheric pressure on Earth. (a) Find the mass of the gas in the balloon. (b) The baby blows an additional mass of 8.00 × 10^{20} kg of helium into the balloon. At the same time, she wanders closer to the Sun and the pressure in the balloon doubles. Find the new temperature inside the balloon, whose volume remains constant.

31. Just 9.00 g of water is placed in a 2.00-L pressure cooker and heated to 500°C. What is the pressure inside the container?

32. One mole of oxygen gas is at a pressure of 6.00 atm and a temperature of 27.0°C. (a) If the gas is heated at constant volume until the pressure triples, what is the final temperature? (b) If the gas is heated until both the pressure and volume are doubled, what is the final temperature?

33. The mass of a hot-air balloon and its cargo (not including the air inside) is 200 kg. The air outside is at 10.0°C and 101 kPa. The volume of the balloon is 400 m^3. To what temperature must the air in the balloon be heated before the balloon will lift off? (Air density at 10.0°C is 1.25 kg/m^3.)

34. Your father and your little brother are confronted with the same puzzle. Your father’s garden sprayer and your brother’s water cannon both have tanks with a capacity of 5.00 L (Figure P19.34). Your father inserts a negligible amount of concentrated insecticide into his tank. They both pour in 4.00 L of water and seal up their tanks, so that they also contain air at atmospheric pressure. Next, each uses a hand-operated piston pump to inject more air, until the absolute pressure in the tank reaches 2.40 atm and it becomes too difficult to move the pump handle. Now each uses his device to spray out water—not air—until the stream becomes feeble, as it does when the pressure in the tank reaches 1.20 atm. Then he must pump it up again, spray again, and so on. In order to spray out all the water, each finds that he must pump up the tank three
times. This is the puzzle: most of the water sprays out as a result of the second pumping. The first and the third pumping-up processes seem just as difficult, but result in a disappointingly small amount of water coming out. Account for this phenomenon.

35. (a) Find the number of moles in one cubic meter of an ideal gas at 20.0°C and atmospheric pressure. (b) For air, Avogadro’s number of molecules has mass 28.9 g. Calculate the mass of one cubic meter of air. Compare the result with the tabulated density of air.

36. The void fraction of a porous medium is the ratio of the void volume to the total volume of the material. The void is the hollow space within the material; it may be filled with a fluid. A cylindrical canister of diameter 2.54 cm and height 20.0 cm is filled with activated carbon having a void fraction of 0.765. Then it is flushed with an ideal gas at 25.0°C and pressure 12.5 atm. How many moles of gas are contained in the cylinder at the end of this process?

37. A cube 10.0 cm on each edge contains air (with equivalent molar mass 28.9 g/mol) at atmospheric pressure and temperature 300 K. Find (a) the mass of the gas, (b) its weight, and (c) the force it exerts on each face of the cube. (d) Comment on the physical reason why such a small sample can exert such a great force.

38. At 25.0 m below the surface of the sea (ρ = 1 025 kg/m³), where the temperature is 5.00°C, a diver exhales an air bubble having a volume of 1.00 cm³. If the surface temperature of the sea is 20.0°C, what is the volume of the bubble just before it breaks the surface?

39. The pressure gauge on a tank registers the gauge pressure, which is the difference between the interior and exterior pressure. When the tank is full of oxygen (O₂), it contains 12.0 kg of the gas at a gauge pressure of 40.0 atm. Determine the mass of oxygen that has been withdrawn from the tank when the pressure reading is 25.0 atm. Assume that the temperature of the tank remains constant.

40. Estimate the mass of the air in your bedroom. State the quantities you take as data and the value you measure or estimate for each.

41. A popular brand of cola contains 6.50 g of carbon dioxide dissolved in 1.00 L of soft drink. If the evaporating carbon dioxide is trapped in a cylinder at 1.00 atm and 20.0°C, what volume does the gas occupy?

42. In state-of-the-art vacuum systems, pressures as low as 10⁻⁹ Pa are being attained. Calculate the number of molecules in a 1.00-m³ vessel at this pressure if the temperature is 27.0°C.

43. A room of volume V contains air having equivalent molar mass M (in g/mol). If the temperature of the room is raised from T₁ to T₂, what mass of air will leave the room? Assume that the air pressure in the room is maintained at P₀.

44. A diving bell in the shape of a cylinder with a height of 2.50 m is closed at the upper end and open at the lower end. The bell is lowered from air into sea water (ρ = 1.025 g/cm³). The air in the bell is initially at 20.0°C. The bell is lowered to a depth (measured to the bottom of the bell) of 45.0 fathoms or 82.3 m. At this depth the water temperature is 4.0°C, and the bell is in thermal equilibrium with the water. (a) How high does sea water rise in the bell? (b) To what minimum pressure must the air in the bell be raised to expel the water that entered?

Additional Problems

45. A student measures the length of a brass rod with a steel tape at 20.0°C. The reading is 95.00 cm. What will the tape indicate for the length of the rod when the rod and the tape are at (a) −15.0°C and (b) 55.0°C?

46. The density of gasoline is 730 kg/m³ at 0°C. Its average coefficient of volume expansion is 9.60 × 10⁻⁴/°C. If 1.00 gal of gasoline occupies 0.00380 m³, how many extra kilograms of gasoline would you get if you bought 10.0 gal of gasoline at 0°C rather than at 20.0°C from a pump that is not temperature compensated?

47. A mercury thermometer is constructed as shown in Figure P19.47. The capillary tube has a diameter of 0.004 00 cm,
and the bulb has a diameter of 0.250 cm. Neglecting the expansion of the glass, find the change in height of the mercury column that occurs with a temperature change of 30.0°C.

48. A liquid with a coefficient of volume expansion $\beta$ just fills a spherical shell of volume $V_i$ at a temperature of $T_i$ (see Fig. P19.47). The shell is made of a material that has an average coefficient of linear expansion $\alpha$. The liquid is free to expand into an open capillary of area $A$ projecting from the top of the sphere. (a) If the temperature increases by $\Delta T$, show that the liquid rises in the capillary by the amount $\Delta h$ given by $\Delta h = (V_i/A)(\beta - 3\alpha)\Delta T$. (b) For a typical system, such as a mercury thermometer, why is it a good approximation to neglect the expansion of the shell?

49. Review problem. An aluminum pipe, 0.655 m long at 20.0°C and open at both ends, is used as a flute. The pipe is cooled to a low temperature but then is filled with air at 20.0°C as soon as you start to play it. After that, by how much does its fundamental frequency change as the metal rises in temperature from 5.00°C to 20.0°C?

50. A cylinder is closed by a piston connected to a spring of constant $2.00 \times 10^5$ N/m (see Fig. P19.50). With the spring relaxed, the cylinder is filled with 5.00 L of gas at a pressure of 1.00 atm and a temperature of 20.0°C. (a) If the piston has a cross-sectional area of 0.010 m² and negligible mass, how high will it rise when the temperature is raised to 250°C? (b) What is the pressure of the gas at 250°C?

51. $\Delta \rho/\rho = -\beta \Delta T$ (a) Show that the fractional change in density for a change in temperature $\Delta T$ is $\Delta \rho/\rho = -\beta \Delta T$. What does the negative sign signify? (b) Fresh water has a maximum density of 1.000 g/cm³ at 4.0°C. At 10.0°C, its density is 0.9997 g/cm³. What is $\beta$ for water over this temperature interval?

52. Long-term space missions require reclamation of the oxygen in the carbon dioxide exhaled by the crew. In one method of reclamation, 1.00 mol of carbon dioxide produces 1.00 mol of oxygen and 1.00 mol of methane as a byproduct. The methane is stored in a tank under pressure and is available to control the attitude of the spacecraft by controlled venting. A single astronaut exhales 1.09 kg of carbon dioxide each day. If the methane generated in the respiration recycling of three astronauts during one week of flight is stored in an originally empty 150-L tank at $-45.0^\circ$C, what is the final pressure in the tank?

53. A vertical cylinder of cross-sectional area $A$ is fitted with a tight-fitting, frictionless piston of mass $m$ (Fig. P19.53). (a) If $n$ moles of an ideal gas are in the cylinder at a temperature of $T$, what is the height $h$ at which the piston is in equilibrium under its own weight? (b) What is the value for $h$ if $n = 0.200$ mol, $T = 400$ K, $A = 0.0080$ m², and $m = 20.0$ kg?

54. A bimetallic strip is made of two ribbons of dissimilar metals bonded together. (a) First assume the strip is originally straight. As they are heated, the metal with the greater average coefficient of expansion expands more than the other, forcing the strip into an arc, with the outer radius having a greater circumference (Fig. P19.54a). Derive an expression for the angle of bending $\theta$ as a function of the initial length of the strips, their average coefficients of linear expansion, the change in temperature, and the separation of the centers of the strips ($\Delta r = r_2 - r_1$). (b) Show that the angle of bending decreases to zero when $\Delta T$ decreases to zero and also when the two average coefficients of expansion become equal. (c) What If? What happens if the strip is cooled? (d) Figure P19.54b shows a compact spiral bimetallic strip in a home thermostat. The equation from part (a) applies to it as well, if $\theta$ is interpreted as the angle of additional bending caused by a change in temperature. The inner end of the spiral strip is fixed, and the outer end is free to move. Assume the metals are bronze and invar, the thickness of the strip is 2 $\Delta r = 0.500$ mm, and the overall length of the spiral strip is 20.0 cm. Find the angle through which the free end of the strip turns when the temperature changes by one Celsius degree. The free end of the strip supports a capsule partly filled with mercury, visible above the strip in Figure P19.54b. When the capsule tilts, the mercury shifts from one end to the other, to make or break an electrical contact switching the furnace on or off.